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METHODS OF INTERFERENCE-FREE RECEPTION OF FREQUENCY MODULATED A--ETC(U)

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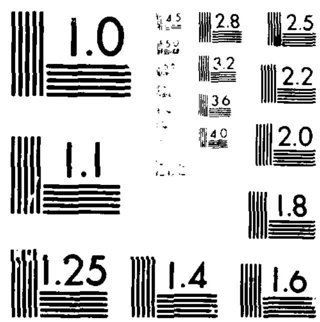

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FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

METHODS OF INTERFERENCE-FREE RECEPTION OF FREQUENCY
MODULATED AND PHASE MODULATED SIGNALS

FOREIGN TECHNOLOGY DIVISION

EDITED TRANSLATION

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

SECTION I

Problems of Interference-Free Reception of FM, PM and AM Signals

A.S. Vinitskiy

Method of Reduced Coordinate Systems as a Means of Reducing Redundancy in the Flow of Scientific Information

It was shown, using a number of examples, that the uses of a coordinate system which has been reduced to a nonsteady state system being studied, can increase the efficiency of heuristic methods of investigation and can reduce many of the newly emerging problems to already solved problems, thereby decreasing the spurious redundancies in scientific investigations.

Mathematical difficulties in the rigorous analysis and engineering calculation of the appropriate devices and systems are characteristic for problems on the interference-free reception and the reception threshold for FM [frequency modulated] and PM [phase modulated] signals. This is associated with the basic nonadditivity of the FM and PM spectra for the signals, with the nonlinearity of the process for filtering the corresponding parameters of the signal, the basic unsteady state of the optimum selective circuits in the systems for their processing, and with a large number of other features of the problem.

During all of the years in which FM and PM have been widely used, an intensive search has been made for different (usually asymptotic) approaches to overcome these difficulties and to set up consistent theory for the optimum reception of FM and PM

signals. However, up to this time it is far from accomplished and many of its segments are the subject of lively debate. The number of publications on this problem is increasing at a vast rate, but, unfortunately, the tempo of the increase greatly exceeds the rate at which their essence is accurately explained.

Such a situation is characteristic not only for the given group of problems but it is related to two basic circumstances: the increase in the number of publications is totally unequal to the increase in the volume of useful information; the universal propagation of the methods of formal-mathematical logic in studying the complex phenomena and objects also has negative consequences, since it leads to a loss of their physical image, which is the basis of heuristic thinking.

We will pause briefly on these two circumstances in connection with the problem of the tracking reception of FM signals.

Recently much has been written about the "information explosion" - the precipitous, exponential increase in the volume of information which is caving in on the scientist. In this case, the number of scientific publications which, as it has been calculated, is doubling every 10-15 years [1], is often taken as the index of its growth [1].

Recently sociological studies have shown clearly [1] that the effectiveness of thw work of the scientist is determined not by the number of published works but mostly by the number of times they are cited in other works. As Dzh. Bernal pointed out long ago, the evaluation of the activity of scholars in terms of the total number of publications is very harmful for science, since in the pursuit of numbers, scientific journals become clogged with immature studies which investigate problems

which were solved long ago albeit in a somewhat changed view. These publications create the spurious redundancy in the total flow of scientific information. The increase at times is taken as an increase in useful information. In this case it is ever more difficult, in the growing flow of publications, to find small nuggets which are authentic new scientific information.

One of the effective media for decreasing such spurious redundancy is the seeking out of "bonds" between related classes of problems which can be used to reduce the newly emerging problem, which at first glance appears to be new and complex, to a simpler problem which has already been solved. The widely used method of electro-mechanical analogies is a clear example of the efficiency of such an approach.

There is one other difficulty in such an approach when applied to the problems of the tracking reception of FM signals. It is related to the fact that, regardless of whether a tracking filter or tracking generator are used in the system, the FM demodulator as a whole is a nonsteady-state system which, as a rule, does not allow it to be considered as quasistable and, therefore, does not allow us to use the well developed methods for the analysis and synthesis of steady-state systems.

It appears that under these conditions a different approach would be an effective means of establishing the "bond" between similar problems for steady and unsteady-state systems, i.e., the use of the method of reduced coordinates. The essence of this method [2] boils down to using intrinsic basic functions and nonisometric, reduced scales for time, amplitude and frequency which are characteristic for the given, complex modulated oscillations or for the given selective system.

The complex-modulated oscillations, which coincide in form with the intrinsic oscillations of the system, degenerate, upon conversion to these reduced coordinates, to harmonic oscillations and selective circuits with variable parameters which are transformed into steady-state circuits[1]. This means that the entire arsenal of mathematical means for studying steady-state systems can be used to study them in the reduced coordinates and, specifically, the relationship can be readily established between like problems for steady-state and nonsteady-state systems. Thus, in [2] it was shown that almost all problems on transition processes in nonsteady-state filters can be reduced to analogous problems for which have already been solved steady-state filters. It was shown in [5] that a whole class of complex polynomial filters of different order and with variable parameters is reduced to an analogous class of polynomial filters with constant parameters, upon conversion to the reduced coordinates, which has already been studied. The number of such examples can be enlarged.

Of course, it is not enough to eliminate the spurious redundancy in the flow of scientific information. In such a case, we must provide for the proper tempo for the forward progress of the growing flow of useful, genuinely new information. We will dwell here on some of the conditions which are necessary for this.

---

[1] The following important circumstance should be pointed out: when it becomes necessary to convert oscillations or systems from absolute coordinates to reduced coordinates or vice versa, the conversion methods may either be precise [3, 4] or approximate [2]; however, the adequacy of the reduced coordinates method for the given nonstationary oscillations or systems comes from the profound physical essence of the studied process and does not depend on the method chosen for the conversion.

Today, much of the work of scientists is directed at the development of rigorous methods of analyzing and synthesizing electronic and cybernetic systems with the constant expansion of the use of computers. These methods have reached a high state of development and flexibility. However,they are all set up for certain, idealized models which reflect a certain state of cognizance of the phenomenon and therefore they can only be used to increase the knowledge of the subject at some level which has already been achieved but cannot go beyond its limits. For example, studies of the threshold phenomena as well as accounting for the effect of wide band FM remain beyond the limits of possibility for the existing methods of statistical synthesis of the optimum demodulators. These methods stimulate the conversion to a new level of knowledge which lies beyong the limits of the given model only in those cases which lead to contradictions or absurdity.

Let us turn our attention to the fact, which is not coincidental that each theory, when it achieves a completed, closed, noncontradictory form, can no longer provide for reaching qualitatively new results. Conversely, each new, fresh theory unavoidably contains elements of heuristics, intuitive synthesis, and as yet unresolved contradictions. But the outlet to new horizons is exactly the overcoming of these very contradictions.

In this connection there must be a harmonic combination of the two aspects of a single process of scientific endeavor: logic (using the entire arsenal of means for its methematical solution) and intuition. The basic role in this, in achieving a new level of perception, is intuition, first of all. Already in the last century one of the leading mathematicians, Poincaré, clearly formulated the relationship between logic and intuition

with respect to mathematics itself. Specifically he wrote: "logic proves, intuition invents. Logic is necessary but by itself it cannot create anything that is actually new... It's basis is passive. Intuition gives its work direction." And further, "Logic...does not show the path to the goal. For this we must see the goal from afar and intuition gives us the ability to see. Without it the geometrician would be like the writer who is faultless in his orthography but who has no ideas." The intuitive way is always much shorter than the path of logical proofs which is characteristic of mathematical methods. Therefore, the effective use of the intuitive approach should provide for an increase in the flow of genuinely new scientific information. But, as recent studies have shown [6], intuition does not operate with binary or other symbols, but rather with whole images. Therefore, in order to make effective use of the intuitive method to "see" the problem's condition as a whole, we must not lose track of its visual, physical image.

This determines the basic role of the analog methods of investigation and the importance of the proper selection of the coordinate system in which the problem is to be studied. The method of reduced coordinates can serve as an effective means for solving the problem in application to the theory of FM signals and nonsteady-state selective systems which are widely used for their optimum reception. In fact, since the reduced coordinates correspond to the nature of the sense of the given nonsteady-state system for as complicated actions as desired, all of the processes will be represented in this system of coordinates with the best physical visualization, in the form that the system itself perceives them. In this case it is important that the modulation, which is wide band in the absolute coordinates, degenerate into narrow band

modulation in coordinates which have been reduced to be almost cophasal with the tracking filter, making possible the simplest spectral analysis and, sometimes, even a quasi-stable study of the basically nonsteady-state systems.

This, specifically, allowed the authors of [7] to solve one of the most difficult problems of the stability of a circuit for self-cophasing a tracking filter by the method of spectral analysis in reduced coordinates. It was shown in [2, 4] that the problem of selecting wide band AM and FM signals with respect to form by using nonsteady-state filters, which is reduced in absolute coordinates to the complex problem of separating signals with overlapping spectra, is reduced to a simple problem of the frequency separation of the signals with nonoverlapping spectra in the reduced coordinates. Conversion to reduced coordinates was used in [2, 8] to carry out a number of difficult problems of studying the threshold, taking into account modulation, by reducing them to problems without modulation with the graphic physical treatment of the results. The effectiveness of the method of reduced coordinates in setting up new models for signals and systems is demonstrated in the article by V.A. Zaytsev, V.I. Manenkov, and O.E. Abramyants which can be found in this collection.

In conclusion, it should be pointed out that the method of reduced coordinates does not exhaust, by any means, the arsenal of methods for both reducing the spurious redundancy in scientific publications or in stimulating heuristic methods of scientific investigations, in its application to the theory of the tracking reception of FM signals and even more to a wide class of other problems. The aim of this article is to call the attention of specialists to this very important

problem and to indicate some possible approaches for solving it.

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Noise Immunity of a System for the Transmission of Continuous Communications

A.G. Zyuko

Relationships are found which can be used to determine the freedom from interference for systems for the transmission of continuous communications with different methods of modulation and for any type of interference.

A tracking receiver [2, 3] is quasioptimal for the transmission of continuous communications. The known methods for lowering the threshold for FM are versions of the practical realization of such a receiver. There is no general theory for analyzing these methods today. We will start from the fact that there are two types of errors in the transmission of continuous communciations: normal, which determine the freedom of the system from interference in the suprathreshold range, and anomalous, which characterize the freedom of the system from interference at a high interference level (in the subthreshold range) [1, 4].

In calculating the normal errors, the receiver is considered to be a linear device. In the region of the threshold, the flow of the anomalous overshoots at the output of the demodulator is assumed to be a steady-state, Poisson flow.

Normal errors. If the signal $A(t,u)$ is distorted by an additive interference $N(t)$ then the signal which is received can be represented in the form

$$X(t) = A(t, u) + N(t). \qquad (1)$$

where $u(t)$ is the continuous transmitted communication with

an a priori distribution p(u). For a low level of interference the communication which is received u*(t) differs only slightly from that which is transmitted u(t). The deviation $\Delta u = u^*(t) - u(t)$ is the random oscillation which determines the interference at the output of the receiver. The energy spectrum (or power of this oscillation) determines the mean square (normal) error in the suprathreshold range. Since Δu is small then the following representation is valid:

$$A(t, v) = A(t, u) + \Delta u A'_u(t, u), \quad (2)$$

where $A'_u(t, u) = \partial A(t, u)/\partial u$.

It is obvious that if there are no distortions in the channel $A(t, v) - A(t, u) = N(t)$.

Then, according to (2) we have

$$\Delta u = N(t)/A'_u(t, u). \quad (3)$$

The energy spectrum for this oscillation (interference at the output of the receiver) is

$$G_u(\omega) = G_N(\omega)/\overline{A'^2_u(t, u)} = K_D^2(\omega) G_N(\omega), \quad (4)$$

where $G_N(\omega)$ is the energy spectrum for the interference at the input of the receiver, $1 \leqslant u(t) \leqslant 1$ |3|; $K_D(\omega) = 1\Big/\sqrt{\overline{A'^2_u(t, u)}}$ is the transmission coefficient of the receiver demodulator. The expressions for $K_D(\omega)$ are given in Table 1 for some types of demodulation.

Table 1

Type of modulation	AM	PM	FM	VIM
$K_D(\omega)$	$\frac{\sqrt{2}}{mA}$	$\frac{\sqrt{2}}{\Delta\varphi A}$	$\frac{\sqrt{2}\omega}{\Delta\omega A}$	$\frac{\sqrt{2k_\Phi}}{A} \frac{\tau_0}{\Delta\tau}$

Here A—is the amplitude of the signal, $\Delta\varphi$, $\Delta\omega$, $\Delta\tau$ – are the maximum deviations for the phase, frequency, and time shift, k_ϕ—is a coefficient which depends on the shape of the pulse.

We note that the expressions given in the Table for $K_D(\omega)$ coincide with the corresponding expressions for the optimum receiver, from Kotel'nikov's point of view [1]. We arrive at the same results from a study of the ordinary methods of reception based on the vector representation of the signal and interference (Fig. 1) which confirms the known position that ordinary methods of reception achieve potential freedom from interference for small interferences.

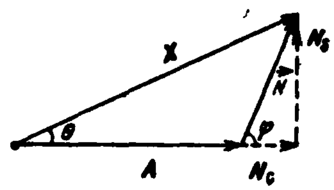

Fig. 1

In addition to a demodulator, a receiver usually contains a filter at the input and a filter at the output of the demodulator which, for low interferences, allow it to be represented as a linear system (Fig. 2) which consists of a bandpass filter (FPCh), a demodulator (D) and a low frequency

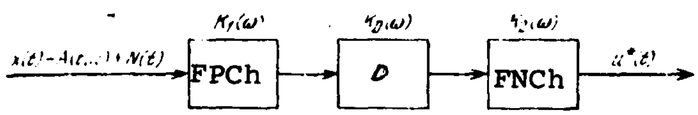

Fig. 2

filter (FNCh) with transmission coefficients of $K_1(\omega)$, $K_D(\omega)$ and $K_2(\omega)$. The energy spectrum for the interference at the output of such a receiver, in the usual case, is determined by the following expression:

$$G_H(\omega) = K_1^2(\omega) K_D^2(\omega) K_2^2(\omega) G_N(\omega). \qquad (5)$$

This is the spectrum for the normal error. The only limitation in the derivation of this equation was that the interference at the input of the receiver must be low as compared with the signal.

The functions $K_1(\omega)$ and $K_2(\omega)$ are determined by the form of the frequency characteristics of the FPCh and FNCh which are used in the receiver.

The energy spectrum for the interference at the output $G_N(\omega)$ is determined by the type of interference. For fluctuation interferences the energy spectrum in the bandpass of the FPCh can be considered as equal to the spectral density $G_N(\omega) = N_0$.

Pulse interferences are the flow of pulses with the random parameters:

$$N(t) = \sum_{k=1}^{n} a_k F\left(\frac{t - t_k}{\tau_k}\right). \qquad (6)$$

In many cases this flow can be considered as steady-state, Poisson flow for which the probability that k pulses will appear in the interval T is determined by the expression

$$P(k) = \frac{(\nu T)^k}{k!} e^{-\nu T}, \qquad (7)$$

where ν is the average number of intereference pulses in a unit of time. The energy spectrum for this flow for an exponential distribution of the intervals between the pulses is determined by the following expression [5]:

where
$$G_N(\omega)=2\nu\overline{a_k^2}H(\omega), \tag{8}$$

$$H(\omega)=\int_0^\infty \tau^2 \mid S(\omega\tau)\mid^2 p(\tau)\,d\tau, \tag{9}$$

$$S(\omega\tau)=\int_{-\infty}^{\infty} F(t)\,e^{-i\omega t}\,dt; \tag{10}$$

$\overline{a_k^2}=a^2+\sigma^2$ is determined by the well known law for the distribution of the interference pulse amplitudes σ^2 is the dispersion, $p(\tau)$ is the distribution in the pulse duration.

If the duration of the pulses is constant $(\tau_k=\tau_0)$, so that $p(\tau)=\delta(\tau-\tau_0)$, then

$$G_N(\omega)=2\nu\overline{a_k^2}\,\tau_0^2 \mid S(\omega\tau_0)\mid^2, \tag{11}$$

In a number of cases the pulses can be considered as similar to δ-pulses $F(t)\approx 2\pi\delta(t-t_k)$ with an energy equal to $a_k\tau_0=2\pi$. Then the interference spectrum at the input will be uniform $\mid S(\omega\tau_0)\mid=1$ and

$$G_N(\omega)=8\pi^2\nu. \tag{12}$$

The interferences in the middle of the spectrum may either have a pulsed nature (signals from foreign radio stations with discrete or pulsed transmissions) or a continuous nature (signals from AM and FM stations with the transmission of continuous communications). The energy spectra for these interferences (modulated signals) are also known in many cases [6].

The energy spectrum for the interference at the output of the receiver, for the action of a group of different interferences, when their overall level is much lower than the signal, can be determined on the basis of the superposition principle as the sum of the energy spectra of the interferences at the output due to the action of the different interferences at the input.

<u>Anomalous errors</u>. For a high level of interferences Δu is large and the linear approximation (2) is no longer valid. Under these conditions the interference scatterings at the output of the receiver cause a reduction in the individual elements (counts) of communication u(t). The concept of a mean square error, in this case, loses its meaning. It is obvious that, in the given case, it is expedient to speak of the transmission of the individual, uncorrelated values of the oscillations in a given range of λ and not the transmission of a continuous oscillation u(t) and to evaluate the freedom from interference in terms of the probability of a reduction in these values (the probability of anomalous errors $P_{\text{ан}}$).

The model for the normal and anomalous errors was suggested by V.A. Kotel'nikov. He also suggested a method for the approximate evaluation of the probability for anomalous errors [1]. This method is based on the replacement of the transmission of continuous communication u(t) with the transmission of discrete, mutually orthogonal, signals which correspond to the discrete elements of this communication. For an even, a priori distribution p(u), these signals can be considered as equally probable. The number of signals m is equal to the number of elements for the resolution of the communication $m' = \lambda/\Delta u_{\text{кор}}$, where u_{kop} is the range

of correlation for the signal A(t, u) in terms of the parameter u. Thus, the problem of determining P_{an} is reduced to the problem of determining the probability of error in the transmission of m equally probable, orthogonal signals. For fluctuation interferences this probability is calculated from the known equations [3]:

$$P_{an} \simeq (m-1)\, V(h) \tag{13}$$

for coherent reception, and

$$P_{an} \simeq \frac{m-1}{2}\, e^{-h^2/2}$$

for incoherent reception

Here

$$V(h) = \frac{2}{\sqrt{2\pi}} \int_h^{\infty} e^{-t^2/2}\, dt, \tag{14}$$

where $h^2 = P_c/P_{in}$ the ratio of the powers of the signal and noise.

In PM and FM reception the anomalous scatterings at the output of the detector, as shown by Rice [7], are due to phase jumps θ (see Fig. 1) for the overall oscillation X(t) for the signal and the interference at the input of the receiver by a value $\theta \geqslant 2\pi$. Consequently, the probability of anomalous errors for FM and PM can be determined as the probability that during the time the signal amplitude U_c is exceeded by the interference U_n, the angle θ will change by a value of $\theta \geqslant 2\pi$:

$$P_{an} = P(U_n > U_c)\, P(\theta \geqslant 2\pi). \tag{15}$$

When the anomalous scatterings (errors) appear relatively infrequently and are a stationary Poisson process, then the average number of anomalous scatterings per second n and the probability of anomalous errors P_{an} in a fixed period of time, for example $T = 1/2F_m$, where F_m is the maximum

frequency in the communication spectrum U(t), are related by the expression

$$P_{\text{ан}} = 1 - e^{-nT} \simeq nT. \qquad (16)$$

According to Rice the number n at the output of the standard ChD [frequency demodulator] for the action of an unmodulated signal on the input and for normal noise is determined by the following expression:

$$n = n_{+} + n_{-} = r\,V(h), \qquad (17)$$

in which r is the inertia radius of the noise spectrum with respect to its central frequency.

In the general case the probability for the anomalous errors can be calculated on the basis of the theory of scatterings [8]. Unfortunately, the statistical description of scatterings for the output effect of the receiver in evaluating the arbitrary oscillations u(t) is lacking as of now.

The anomalous scatterings at the output for the action of pulsed interferences on an FM receiver are also due to phase jumps for the given oscillation by 2π. The probability of anomalous errors in this case is determined, usually, on the basis of expression (15). In [9] this probability was determined both for constant and random amplitudes for the pulsed interference for different FPCh characteristics. Thus, for a hyperbolic law for the amplitude distribution of the interference:

$$p(U_{\text{п}}) = \begin{cases} 2\gamma\, U_{\text{по}}^{2\gamma} / U_{\text{п}}^{2\gamma+1}, & U_{\text{п}} > U_{\text{по}}, \\ 0, & U_{\text{п}} < U_{\text{по}}, \end{cases} \qquad (18)$$

where γ is the distribution parameter. The probability of anomalous errors for a FNCh with gaussian characteristics is equal to

$$P_{ан} = \frac{n\,\Delta f}{2F_{э}\sqrt{\gamma}}\left[1 - 2V\left(\frac{F_{э}\sqrt{\pi\gamma}}{\lambda\,\Delta f}\right)\right], \qquad (19)$$

where $F_{э}$ is the effective pass band for the FPCh and Δf is the deviation in the signal's frequency.

The average number of anomalous scatterings in this case is

$$n = \nu P_{ан}, \qquad (20)$$

The determination of the probability of anomalous errors for the action of a number of different interferences is fraught with great mathematical difficulties. In those cases for which the distribution of the group of interferences can be considered as normal (Gaussian), the problem is reduced to that discussed above for the fluctuation interferences. In the other cases the result obtained for the Gaussian approximation will be the upper limit of the freedom from interference.

<u>Energy spectrum of the interferences at the receiver's output</u>. For an approximate evaluation of the receiver's freedom from interference for a high level of interference and to set up the threshold curves in the generally accepted coordinates, the energy spectrum of the anomalous scatterings can be determined at the output of the receiver. In many cases, the flow of the anomalous scatterings can be considered as stationary, Poisson flow $F_2(t)$ with a duration distribution $p_2(\tau)$ or any type of interference at the input for the receiver. The energy spectrum for this type of flow is determined

on the basis of expressions (8) — (11) if the corresponding parameters for the anomalous scatterings are substituted in them. According to (8) and (11) we have

$$G_{aH}(\omega)=2n\overline{a_k^2}\,H(\omega)\,K_2^2(\omega)\,; \qquad (21)$$

or for $\tau_k=\tau_0=\mathrm{const}$

$$G_{aH}(\omega)=2\pi\overline{a_k^2}\,\tau_0\,|\,S(\omega\tau_0)\,|^2\,K_2^2(\omega), \qquad (22)$$

where $H(\omega)$ and $S(\omega)$ are determined by relationship (9) and (10) for a flow of the anomalous pulses $F_2(t)$, $K_2(\omega)$ is the transmission coefficient of the FNCh. In the range of low frequencies, the amplitude spectrum for the anomalous scatterings can be considered to be even, i.e., the flow of the anomalous scatterings can be considered to be a flow of δ-pulses with an energy equal to 2π. The energy spectrum in this case, according to (12) is determined by the expression

$$G_{aH}(\omega)=8\pi^2 n\,K_2^2(\omega). \qquad (23)$$

The appearance of normal and anomalous errors are incompatible events: in one case there is a slight shift in the values of received communication u*(t) in the vicinity of the true values of the transmitted communication u(t) and in the other case there is a reduction in the individual elements of the communication. Therefore, the energy spectrum for the interference at the output of the receiver for any level of interference can be determined as the sum of the energy spectra for the normal $G_H(\omega)$ and for the anomalous $G_{aH}(\omega)$ taking into account the probability of their appearance:

$$G(\omega)=(1-P_{aH})\,G_H(\omega)+P_{aH}\,G_{aH}(\omega). \qquad (24)$$

By substituting expressions (5) and (21) for $G_H(\omega)$ and $G_{aH}(\omega)$ we find

$$G(\omega)=(1-P_{aн})K_1^2(\omega)K_D^2(\omega)K_2^2(\omega)G_N(\omega)+ + 2n\overline{a_h^2}H(\omega)K_2^2(\omega). \quad (25)$$

Further, the power of the interference at the receiver's output, $P_{п\ вых}$ can be determined and the curves for $(P_c/P_п)_{вых} = f(P_{c\ вх}/P_{п\ вх})$. can be constructed. As the signal/interference ratio increases, the probability for anomalous errors rapidly decreases. Starting with some (threshold) value for the signal/interference ratio at the input, the resulting signal/noise ratio at the output of the receiver is determined only by the normal error. For small values of the signal/interference ratio, the main role in the output effect is that played by the anomalous errors.

As an example, let us determine the energy spectrum for the interference at the output of an FM receiver for the action of a pulse interference on its input. According to Table 1, $K_D^2(\omega)=2\omega^2/\Delta\omega^2A^2$ for FM. The frequency characteristics for the FPCh and FNCh will be considered to be rectangular $K_1^2(\omega) = 1$ and $K_2^2(\omega) = 1$. Then, on the basis of expressions (25), (11) and (22) for low enough frequencies $\omega \ll 2\pi F_в$ (the case in which the modulation index is $\beta \gg 1$) we will have

$$G(\omega)=m\,\omega^2\,\overline{q^2}/\Delta\omega^2F_в^2+2\pi^2\overline{n}, \quad (26)$$

in which $\overline{q^2}=(U_п/A)^2$ is the average value of the square of the maximum values of the interference amplitude $U_п=2a_k\tau_0S(\omega\tau_0)F_в$ at the output of the FPCh to the amplitude of the signal A; $m = \nu P(q < 1)$ is the average number of interference pulses which do not exceed the signal amplitude at the output of

the FPCh in a unit of time; $\bar{n}$ is the average number of anomalous scatterings in a unit of time at the output of the frequency detector. For a communication normalized to a unit interval u(t)

$$\bar{n} = \int_{-1}^{1} n\, p(u)\, du = 2\int_{0}^{1} \nu P_{\text{вн}}\, p(u)\, du. \qquad (27)$$

Thus, as for fluctuation interference, the energy spectrum for the pulsed interference at the output of an FM receiver in the range of low frequencies is the sum of two components: a quadratic and equivalent component.

The power at the outlet of the FNCh, according to (26) will be equal to

$$P_{\text{п}} = \int_{0}^{F_m} G(2\pi f)\, df = \frac{m\,\overline{q^2}\, F_m^3}{3\,\Delta f^2\, F_{\text{э}}^2} + 8\pi^2 \bar{n} F_m. \qquad (28)$$

The power of the received communication $P_C = k_{\text{п}}^{-2}$ where $k_{\text{п}}$ is the peak factor for the communication u(t). For sinusoidal modulation $k_{\text{п}} = \sqrt{2}$, т. е. $P_{\text{c}} = 1/2$.

Tracking reception. The higher freedom from interference of the tracking demodulator (SD) near the threshold is due to its capacity to suppress anomalous scatterings. Studies of the tracking reception of FM signals showed that for each demodulator there is a dertain duration $\tau_{\text{кр}}$ for the threshold phase jump (a jump of $\pm 2\pi$). For $\tau < \tau_{\text{кр}}$ the jump is suppressed but for $\tau > \tau_{\text{кр}}$ the jumps pass through the demodulator, causing anomalous voltage pulses at the output [10]. The value of the critical duration can be readily determined experimentally. If the probability distribution density is known for the duration of the anomalous scatterings p(τ) then the probability can be determined for the appearance

of the threshold scatterings which have a duration of $\tau > \tau_{\text{кр}}$:

$$P_1(\tau_{\text{кр}}) = \int_{\tau_{\text{кр}}}^{\infty} p(\tau)\, d\tau. \qquad (29)$$

Knowing the number of anomalous scatterings n at the input to the demodulator, we will determine the number of anomalous scatterings which pass through the tracking demodulator

$$n_1 = n P_1(\tau_{\text{кр}}). \qquad (30)$$

The energy spectrum and the power of the anomalous scatterings at the output of the SD is determined by means of the formulas which were found above for the standard demodulator if n_1 is used in place of n.

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Optimum Evaluation of the Oscillation Frequency and Tracking Demodulators for FM Signals

Ya. G. Rodionov

It is shown in the article that a tracking filter (SF), a demodulator with a negative frequency feedback (OSCh) and a demodulator of automatic fine tuning of the frequency type (FAPCh) are quasioptimum systems for frequency evaluation. It is suggested that the threshold criterion be used to compare the freedom from interference of the quasioptimum systems for FM reception. Expressions are given for finding the noise bands and the signal/noise ratios for SF, OSCh, and FAPCh systems. It is shown that under real conditions tracking modulators lower the threshold by 2-5 dB as compared with standard demodulators, but the properties of the "hypothetical receiver", which makes use of all of the possible information fed into the FM signals are not fully realized.

The oscillations y(t) at the input to a receiver with optimum frequency evaluation under noise conditions is an additive mixture of the narrow band signal $s(t) = A(t)\cos[\omega t + \varphi(t)]$, (in which ω is an essential parameter, A, φ are non-essential parameters) and a stationary Gaussian noise n(t) with a zero average with a correlation function $k(t) = \delta(\tau) N_0/2$ and a functional probability of

$$w[n(t)] = \mathrm{const}\exp\left[-\frac{1}{N_0}\int_0^T n^2(t)\,dt\right],$$

in which $N_0/2$ is the spectral density of the noise's power,

$\delta(\tau)$ is the delta function, and T is the observation range above the accepted mixture.

Taking into account the assumed slowness for the change in the signal's parameters, the statistical independence of the a priori distributions A, φ, and ω $(w_{pr}(\varphi)=1/2\pi)$, $A(t)$ is a constant value of a known function of time $A_0(t)$ because of which $w_{pr}(A)=\delta(A-A_0)$. For $\varphi T \gg 1$ we can find the aposteriori probability density [1, 2]:

$$w_{ar}(\omega) = \text{const}\, w_{pr}(\omega) I_0\left(\frac{-2}{N_0} z(\omega)\right). \qquad (1)$$

for which $I_0(x)$ is a modified, zero-order Bessel function: $z(\omega)=\sqrt{X^2+Y^2}$;

$$X=\int_0^T y(t)A_0(t)\cos\omega t\,dt;$$

$$Y=\int_0^T y(t)A_0(t)\sin\omega t\,dt. \qquad (2)$$

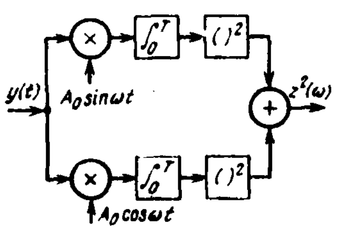

Fig. 1

According to (2), the overall functional scheme for the optimum frequency measuring device must include the device shown in Fig. 1.

According to another quasi-linear method for treating a mixture of signal and noise [3], the optimum instrument for measuring the frequency for a Gaussian a priori density $w_{pr}(\omega)$ can be found from the expression

$$\omega^*(t)=\int_{t-T}^{t} G(t,\tau)b(\tau)d\tau - \overline{\omega^*(t)}, \qquad (3)$$

in which $\overline{\omega^*(t)}$ is the average value of the informative function which is assumed to be known and $b(\tau)$ is a function which is determined by the expression

$$\int_{t-\Delta}^{t} b(\tau)\, d\tau = \frac{\partial}{\partial \lambda} \ln w_{ps}(y/\lambda^*); \qquad (4)$$

Δ is one of the integrands into which the (t-T, t) interval is divided for the sampling which is needed in analyzing the given process, λ is the informative parameter, $w_{ps}(y/\lambda^*)$ is the probability function for the process y(t) in the given integrand. The quantity Δ is much larger than the interference correlation interval but much smaller than the correlation interval for the informative function $\lambda(t)$.

The function $G(t, \tau)$ is the pulsed response of the linear system:

$$G(t, \tau) = C(t, \tau) + k \int_{t-T}^{t} C(t, s) G(t, s)\, ds, \qquad (5)$$

for which

$$C(t, \tau) + k \int_{t-T}^{t} C(t, s) R(s, \tau)\, ds = R(t, \tau), \qquad (6)$$

where $R(t, \tau)$ is the correlation function for the signal y(t), and k is the mean value of the function K(t) determined for the integrand Δ in the form

$$\int_{t-\Delta}^{t} K(t)\, dt = \frac{\partial^2}{\partial \lambda^2} \ln w_{ps}(y/\lambda). \qquad (7)$$

The optimum structural scheme which corresponds to expression (3) is shown in Fig. 2. Here 1 is the nonlinear connection (discriminator) which separates the function b(t); 2 is the linear filter with a pulsed reaction $G(t, \tau)$; 3 is the summation device. The filter's output response

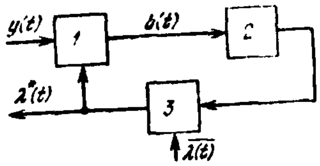

Fig. 2

is added to the mathematical expectation for the function $\lambda(t)$ and, as a result, the approximate value of $\lambda^*(t)$ is obtained.

The practical realization of the processing algorithms (1)-(3) can be achieved in various ways [4-8].

Tracking demodulators can also be included among the quasioptimum systems for receiving FM signal analogs. We will demonstrate this using a system of automatic fine tuning of the phase frequency (FAPCh). It follows from expression (4) that the output response b(t) for a nonlinear discriminator in the optimum system (Fig. 2) is determined by the probability function which, in the integrand Δ is written in the form

$$w_{ps}(y/\lambda)=\text{const}\exp\left\{-\frac{1}{N_0}\int_{t-\Delta}^{t}[y(t)-s(t)]^2\,dt\right\}. \quad (8)$$

By converting (8) to the logarithmic form, collecting the derivatives with respect to t, and equating it to expression (4) and assuming that A_0 = const, φ = const, we find

$$b(t)=\frac{\partial}{\partial\lambda}\left\{-\frac{1}{N_0}[y(t)A_0\cos(\omega t+\varphi)]^2\right\}_{\lambda=\lambda^*}. \quad (9)$$

Assuming that FM is an integral type of modulation we differentiate (9) with respect to the parameter $\lambda=\int\omega dt$ и, and by discarding the vibration terms with a doubled value of the frequency ω, we get

$$b(t)\approx-\frac{2}{N_0}\,y(t)A_0\sin(\omega^*t+\varphi). \quad (10)$$

Consequently, the nonlinear discriminator 1 (Fig. 2) in this case must be the amplifier and generator of the supporting voltage. By replacing the optimum linear filter (2) with an ordinary low frequency filter (for example a proportional-integrating component type) and, considering that the approximate value of the parameter $\lambda^*(t)$ is reflected directly in the output response of this filter, we arrive at a system for the quasicoherent treatment of the phases of the FAPCh type.

A tracking demodulator with a frequency feedback system (OSCh) includes in its makeup a multiplier of the signals which are received and the supporting signal (mixer with a narrow band filter of intermediate frequency connected at its output, i.e., the operations inherent in a correlation receiver are carried out. The tracking filter (SF) with the controlled resonance frequency is a device in which the group of optimum, coordinated filters is replaced by a single, quasioptimum filter which controls the approximate value of the output parameter.

For comparing the quasioptimum systems for evaluating the frequency under conditions of high noise levels, it is convenient to carry out the threshold criterion for FM systems. By the threshold of FM system, we mean the minimum possible signal/noise ratio ρ_0 at the input to the receiver taken as twice the infomative band $2F_B$ for the FM signal for which the linearity of the interference-free characteristic $\rho_{вых} = \varphi(\rho_0)$ for FM reception breaks down. Let us designate this threshold ratio by $\rho_{0\pi}$. The lower the $\rho_{0\pi}$, the better, obviously, the threshold properties of the receiver.

For a standard demodulator the threshold ratio $\rho_{0\Pi}$ for a deviation of the characteristic of 1 dB from linearity can be evaluated with sufficient accuracy from the empirical expression [9]:

$$\rho_{0\,\Pi} \approx \frac{B}{2F_{\text{в}}}\left(3{,}9+4{,}8\lg\frac{B}{2F_{\text{в}}}\right), \tag{11}$$

in which B is the noise band of the linear part of the receiver (up to the frequency detector)

In tracking demodulators the noise band B should be calculated taking into account the closed circuit for reverse control. An analysis shows that for SF the value of B in the suprathreshold and threshold regions can be determined approximately by means of the linearized model for the system using equivalent frequency characteristics (EChKh) [10]. It is obvious that the conclusion can also be extended to other types of tracking demodulators. The general expression for the noise band B in these systems is

$$B=2F_{\text{в}}\int_0^{\infty}|K_{\text{ч}}(a)|^2\,da, \tag{12}$$

where $|K_{\text{ч}}(a)|$ is the EChKh modulus, $a = F/F_B$, and F is the modulation frequency.

In a FAPCh system with a proportional-integrating filter the EChKh has the form

$$|K_{\text{ч}}(j\Omega)|=\sqrt{\frac{1+\Omega^2\tau^2}{\left(1-\frac{\tau}{\Delta_c}\Omega^2\right)^2+\Omega^2\left(\frac{1}{\Delta_c}+\tau_2\right)^2}}, \tag{13}$$

where Δ_c is a parameter which is proportional to the system's holding band $(\Delta_c=\Delta\omega_y\,|\sin\varphi_0|=2/\tau_c;\ |\sin\varphi_0|$ is the linearization

parameter), τ, τ_2 are parameters of the proportional-integrating filter.

By calculating the integral $\frac{1}{\pi}\int_0^\infty |K_ч|^2 d\Omega$ we arrive at the expression

$$B_{\phi пч} = \frac{\Delta_c}{2}\frac{1+\Delta_c\tau_2^2/\tau}{1+\Delta_c\tau_2}. \qquad (14)$$

The value of $B_{\phi пч}$ was calculated in a somewhat different form in [11]. Formula (14) is valid for conditions realized in practice

$$2\sqrt{\Delta_c\tau} - 1 \geqslant \Delta_c\tau_2; \quad \Delta_c\tau > 1/4. \qquad (15)$$

Analogous calculations for SF with an integrating control circuit leads to the expression

$$|K_ч(a)| = \sqrt{\frac{1+\lambda^2 a^2}{1+\lambda\left(\lambda-\frac{2}{b}\right)a^2+\frac{\lambda^2}{b^2}a^4}}, \qquad (16)$$

where $b = \Delta F/2F_B$, $\lambda = 2\pi F_B\tau$, ΔF is the band for the uncontrolled circuit of the SF at the level 0.707, and τ is the time constant for the integrating circuit.

For $\alpha\tau \leqslant 4$ $(\alpha=\pi\Delta F)$, which in practice is usually true

$$B_{c\phi} = (\alpha\tau+1)/2\tau. \qquad (17)$$

In an OSCh system with an integrating filter in the control circuit, we find

$$|K_ч(j\Omega)| = \frac{\alpha K}{\sqrt{[\alpha(1+K)-\Omega^2\tau]^2+(1+\alpha\tau)^2\Omega^2}}, \qquad (18)$$

where K is the amplification coefficient for the feedback loop and α is the attenuation coefficient for the UPCh

narrow band filter.

Determining the noise band from the expression

$$B=\frac{(1+K)^2}{K^2}\int_0^\infty |K_ч(j\Omega)|^2 d\Omega$$

gives

$$B_{осч}=\alpha(1+K)/2(1+\alpha\tau). \qquad (19)$$

Formula (19) is valid for $(1+\alpha\tau)^2/\alpha\tau\leqslant 4(1+K)$, i.e., for a large amplification K in the feedback loop.

In order to evaluate the threshold properties of the given systems it is necessary to equate their noise bands. For SF and OSCh the same values of τ can be chosen directly for a given α. In this case the amplification K in the OSCh system is determined from the expression

$$K=(1+\alpha\tau)^2/2\alpha\tau-1. \qquad (20)$$

The noise band in the FAPCh system is a function of a number of parameters. We can find the required value of the parameter Δ_c from equalities (14) and (17):

$$\Delta_c=\frac{2}{\tau_2}\left[\frac{\alpha\tau}{4}\left(1+\frac{1}{\alpha\tau}-\frac{1}{\alpha\tau_2}\right)+\right.$$
$$\left.+\sqrt{\frac{\alpha^2\tau^2}{16}\left(1+\frac{1}{\alpha\tau}-\frac{1}{\alpha\tau_2}\right)^2+\frac{\alpha\tau}{4}+\frac{1}{4}}\right]. \qquad (21)$$

If, for example, we choose $\tau_2/\tau=0{,}1$, $F_в=15$ kHz $|\sin\varphi_0|=0{,}2$ $\tau=3/\alpha=3{,}2\cdot10^{-5}$ c,

then we will have $\Delta\omega_y=0{,}94\cdot10^6$ rad/sec $\tau=0{,}32\cdot10^{-5}$. In this case the noise band for the FAPCh system is equal to the noise

band for the SF at $\alpha\tau = 3$. If it is assumed that $\alpha\tau = 1$ in the SF then its noise band will be equal to the noise band for the FAPCh system with the same values of τ_2/τ_1, $|\sin\varphi_0|$ for $\tau = 1/\alpha = 1{,}06 \cdot 10^{-5}$ c, $\Delta\omega_y = 11{,}3 \cdot 10^5$ rad/sec, $\tau_c = 0{,}89 \times$ $\times 10^{-5}$ sec, $\tau_2 = 0{,}0106 \cdot 10^{-5}$ sec.

If we take into account the fact that the noise bands for the three systems are equal we can find the overall threshold ratio for them ρ_{0n}, from (11). For $\alpha\tau = 3 (B = 62{,}5 \cdot 10^3$ Hz it is equal to 10.5 dB and for $\alpha\tau = 1 (B = 94 \cdot 10^3$ Hz) it is equal to 13 dB.

The values that were found for $\rho_{0\pi}$ can be compared with the threshold ratios for other systems for FM reception and, specifically, with the limiting threshold ratio which is intrinsic to some "hypothetical receiver" which has all of the information properties of the FM signal [12, 13].

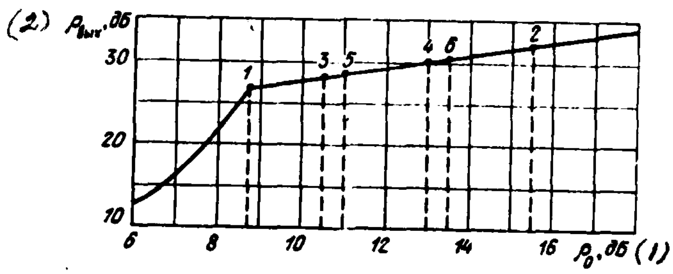

Fig. 3

Key: (1) dB; (2) R_{vykh}

The curve for the freedom from interference is shown in Fig. 3 for FM reception with an index m = 5. The points and ordinates are marked on it which correspond to the threshold

ratio $\rho_{0п}$ for different types of receivers. Point 1 gives the limiting threshold ratio (on the order of 8.8 dB) which is realized by the "hypothetical receiver" and point 2 determines the threshold properties of the standard FM demodulator with a noise band B = 150 kHz. Points 3 and 4 refer to a tracking demodulator with $\alpha\tau = 3$ and 1, respectively. Point 5 gives the threshold for the correlation receiver [4] and point 6 gives the threshold for a system with a divided band [6]. As we can see from this figure, tracking demodulators in a system [4,6] lower the threshold by 2-5 dB as compared with a standard demodulator. In addition they are not yet totally near the "hypothetical receiver" which indicates that it is possible to improve the threshold properties of quasioptimum systems of FM reception still further.

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Analysis of the Interference Protection for an FM Demodulator, With Subtraction of the Deviations for a Signal Modulated by an Effective Communication

I.L. Papernov

The effect of modulation was determined on the interference protection of an FM demodulator subtracting the deviations. It was shown that the spurious amplitude modulation which arises in a narrow band filter is the determining factor.

An analysis of the freedom from interference for a transducer with subtraction of the deviations (PVD) whose structural scheme is shown in Fig. 1, was made in [1-3] without taking the modulation into account.

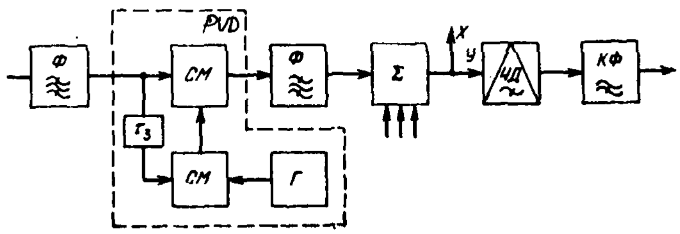

Fig. 1

For modulation the cophasal and orthogonal projection of the noise vector on the signal vector depends on the transmitted communication. In addition, the FM signal, upon passing through the filter, acquires a spurious amplitude modulation (PAM).

The first effect was studied in [4] on the basis of the pulse model for the threshold mechanism of FM ("method

of anomalous pulses"). The second effect was studied in [5] on the basis of the method of weighed probabilities.

We will take both effects into consideration and we will find the mean number of anomalous pulses $N_{\Delta\omega}$ [4] with a static detuning $\Delta\omega$ of the carrier from the central frequency of the filter. The signal's amplitude in this case is taken to be equal to $U_c(\Delta\omega) = U_c K(\Delta\omega)$ assuming that $\Delta\omega$ changes slowly as compared with the filter band. We will average $N_{\Delta\omega}$ with respect to $\Delta\omega$:

$$N(\rho) = \int N_{\Delta\omega} W(\Delta\omega)\, d\Delta\omega =$$
$$= \frac{2r}{\Delta\omega_{ш}} \int_0^\infty \sqrt{\frac{2}{\pi}}\, e^{-t^2} \{ \sqrt{1+gt^2} \times$$
$$\times [1 - \Phi(\sqrt{(1+gt^2) K^2(\Delta\omega_э t)\rho})] +$$
$$+ t\sqrt{g} \exp\{-\rho K^2(\Delta\omega_э t)\}\, \Phi(\sqrt{g t^2 \rho K^2(\Delta\omega_э t)})\}\, dt.$$

Here $N(\rho)$ is the total number of positive and negative pulses referred to half of the filter's noise band, where ρ is the signal/noise ratio at the input of the ChD; r is the radius of inertia of the square of the AChKh (amplitude frequency characteristic) for the filter $K^2(\Delta\omega)$, $\Delta\omega_ш$ is the noise band of the filter, $\Delta\omega_э$ is the effective frequency deviation, and $g = \Delta\omega_э^2 / r^2$ is a parameter which takes into account the dependence of the cophasal and orthogonal projections of the noise on the signal vector on the modulation.

Let us consider two of the AChKh filters which are most often encountered at the input to a ChD (frequency detector): a Gaussian AChKh which corresponds to a multicircuit filter

$$K^2(\Delta\omega) = \exp(-\Delta\omega^2/\beta^2);\ \Delta\omega_ш = \beta\sqrt{\pi},\ r = \beta,$$

and the AChKh of a two-circuit filter with a critical circuit

$$K^2(\Delta\omega) = \frac{1}{1+(\Delta\omega/\Delta\omega_{0,7})^4}; \quad \Delta\omega_{ш} = \frac{\pi}{\sqrt{2}}\Delta\omega_{0,7}, \; r = \Delta\omega_{0,7}.$$

The results of calculating the dependence N(ρ) for some values of the parameter $q = \Delta\omega_s^2/\Delta\omega_{0,7}^2$ are given by the solid lines in Fig. 2 for the Gaussian filter and in Fig. 3 for the two-circuit filter. The dependences taking into account only PAM (q = 0) are shown by the dotted lines. As we can see from the graphs, the PAM has the most effect on the increase in the number of pulses for Gaussian modulation.

If the spectral density of the anomalous noise in the Rice method is determined by the mean number of pulses N, then in the method of weighted probabilities it is characterized by the value $(1.85/2\pi)p$, where $p = P(U_ш > U_C)$ is the probability that the noise amplitude will exceed the signal's amplitude.

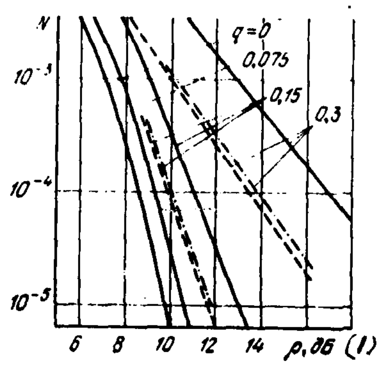

Fig. 2

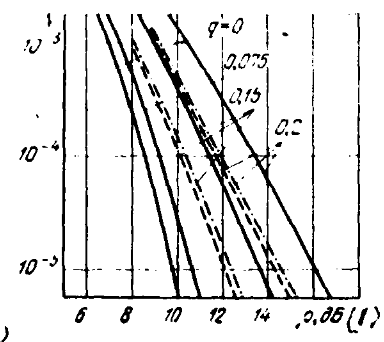

Fig. 3

Key: (1) dB

By taking into account the PAM of the signal at the input to the ChD we have [5]

$$p = \int_{-\infty}^{+\infty} P\{U_{ш} > U_c K(\Delta\omega)\} W(\Delta\omega)\, d\Delta\omega =$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \exp\{-\rho K^2(\Delta\omega_{э}, t)\}\, e^{-t^2/2}\, dt.$$

We can see from the curves for the dependence of the quantity (1.85/2)p on ρ, , which are shown in Figs. 2 and 3 by the dotted lines, that if the effect of the PAM is taken into account both methods give similar results.

On the whole, for the action of modulation and for $q \leqslant 0.1$, the freedom from interference for FM reception deteriorates no more than by 1 dB; the deterioration increasing with an increase in q.

The effective deviation in the frequency at the output of PVD of a transducer with subtraction of the deviations (PVD) is related to the effective deviation of the frequency at the input to the device $\Delta\omega_{\text{эф вх}}$ by means of the relationship [2]

$$\Delta\omega^2_{\text{эф вых}} = 2\Delta\omega^2_{\text{эф вх}}\left[1 - \frac{\sin\Omega_{в}\tau_{з} - \sin\Omega_{н}\tau_{з}}{(\Omega_{в} - \Omega_{н})\tau_{з}}\right], \quad (1)$$

where $\tau_{з}$ is the delay time in the delay line, Ω_B and Ω_H are the upper and lower frequencies of the group modulation spectrum.

It is expedient to use a standard ChD followed by a corrective filter KF in the group channel

$$B^2_{к}(\omega) = [4\sin^2(\omega\tau_{з}/2)]^{-1}.$$

as a demodulator.

The energy spectrum of the noise phase was found in [2] at the output of the PVD:

$$G_\theta(\omega) = \frac{1}{a\rho_{\text{вх}}}\left[K^2(\omega)\,4\sin^2\frac{\omega\tau_з}{2} + \frac{K\,0{,}75}{\rho_{\text{вх}}}\right]\frac{1}{\Delta\omega_{\text{ш вх}}}. \quad (2)$$

Here K is the number of dispersion phases, $a = \Delta\omega_{\text{ш вх}}/\Delta\omega_{\text{ш}}$; $\rho_{\text{вх}} = \sum_1^k U^2_{c\,\text{вх}\,i}/P_{\text{ш вх}}$; $P_{\text{ш вх}}$ is the power of the noise at the input to one receiver.

Although in the general case the distribution for the cophasal X and orthogonal Y projections of the noise at the output of the PVD differ from the normal distribution, this difference is insignificant in the FM threshold region for d > 5 and $\tau_з\beta_{\text{вх}} > 1$

Expressions were found for $\rho_{\text{вых}}$ in [2] (signal/noise ratio at the output to the PVD) and α (ratio of the power for the orthogonal projection of the noise to the power for the cophasal projection). These expressions have the following form for a signal amplitude equal to one:

$$\overline{X}^2 + \overline{Y}^2 = \frac{1}{\rho_{\text{вых}}} = \frac{1}{a\rho_{\text{вх}}}\left(2 + \frac{0{,}75\,K}{\rho_{\text{вх}}}\right); \quad (3)$$

$$\alpha = Y^2/X^2 = (1-\gamma)/(1+\gamma),\ \gamma = I/(1 + 0{,}75K/2\rho_{\text{вх}}); \quad (4)$$

$$I = \begin{cases} \exp(-\beta^2\tau_з^2/4) \text{ — in the first case} \\ \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\Delta\omega_{0,7}\tau_з}{\sqrt{2}}\right)\times \\ \times\exp\left(-\frac{\Delta\omega_{0,7}\tau_з}{\sqrt{2}}\right) \text{ — in the second case} \end{cases}$$

It was found in [6] that $P(U_{\text{ш}} > U_c)$ for $\alpha \neq 1$. The following approximation of the rather complex equations in [6] is permissible for $4 \leqslant \rho_{\text{вых}} \leqslant 16$ and $0{,}3 \leqslant \alpha \leqslant 1$

$$p = P(U_{\text{ш}} > U_c) = \exp[-C(\alpha)\rho_{\text{вых}}],$$

in which

$$C(\alpha) = 0{,}69 + 0{,}31\alpha + 0{,}375(\alpha^2 - \alpha^3),$$

For $\alpha < 0,3$

$$P = \frac{1}{\sqrt{1-\alpha}}\left[1 - \Phi\sqrt{\frac{1+\alpha}{2}\rho_{\text{вых}}}\right].$$

Thus, in analyzing the threshold by the method of weighted probabilities we get the following formula for the power of the noises in milliwatts in a telephone channel with a frequency ω_K and a band width $\Delta\Omega_K$ at the point of zero level of measurement

$$P_{\text{шт}} = \frac{K^2_{\text{пс}}\,\Delta\,\Omega_{\text{к}}\,\Delta\,\omega_{\text{ш}}\,B^2_{\text{к}}(\omega)}{\Delta\,\omega^2_{\text{к вх}}}\left[\frac{1,85}{2\pi}P + \frac{\omega^2_{\text{к}}}{\Delta\,\omega_{\text{ш}}}\,G_\theta(\omega)\right], \quad (5)$$

where $K^2_{\text{пс}}$ is the psophometric coefficient and $\Delta\omega_{K\ BX}$ is the deviation in the frequency which corresponds to the measuring level at the input to the PVD.

Since the projection of the noise on the output of the PVD, for the values of τ_2 which are used in practice, are approximately normal and are independent, then the number of pulses N at the output of the device is determined by the method in [4] by substituting the probability that the signal's amplitude will be exceeded by the cophasal component of the noise at the output to the PVD. This means that

$$\rho' = \rho_{\text{вых}}(1+\alpha)/2. \quad (6)$$

should be used in place of ρ in the appropriate formulas and graphs.

In the experimental study of the effect of modulation on the freedom from interference for a transducer the PVD had the following parameters: $\Delta\omega_{max}/2\pi=10$ MHz; $2\Delta\omega_{0,7}/2\pi=1,5$ MHz; $\tau_{з}=1\mu$sec; $K=1$ (single reception $\Delta\omega_{к\,вх}/2\pi=100$ kHz; $\omega_{к}/2\pi=112$ kHz; $\Delta\Omega_{к}/2\pi=$ $=3,1$ kHz. Two-circuit filters with a critical coupling were used at the input and output of the PVD. The psophometric coefficient was not taken into account and no preselection was introduced. The modulation was imitated by white noise which has the spectrum 12-252 kHz and a load level of 9 dB ($\Delta\omega_{эф\,вх}=2,82\,\Delta\omega_{к\,вх}$) which corresponds to 60 telephone channels.

In constructing the graphs, which are shown in Fig. 4 by the dotted lines (for modulation (1) and without modulation (2)), the power of the transitional noises which is equal to 15,000 pW is subtracted from the experimental measurement. The curves which were calculated by using formulas (1)-(6) and using the graphs in Fig. 3 are shown by the solid lines. The shift in the threshold which was found experimentally $\Delta_{э}$ and by calculation Δ_{p} is equal to 1.5 dB. The agreement between the experimental and calculated values is satisfactory.

In conclusion we note that when there is no correction for the phase frequency characteristics for the filter at the output of the PVD, lower deviations are used [$\Delta\omega_{к\,вх}/2\pi=$ (50-70) kHz] to decrease the power of the transitional interferences. In this case, modulation has no effect on the freedom from interference. However, correction of the phase frequency characteristics greatly decreases the transitional interferences. As shown in [7], for ideal correction the

power of the transitional interferences is approximately 200 pW for the parameters used above.

In this case, the effect of modulation must be taken into consideration.

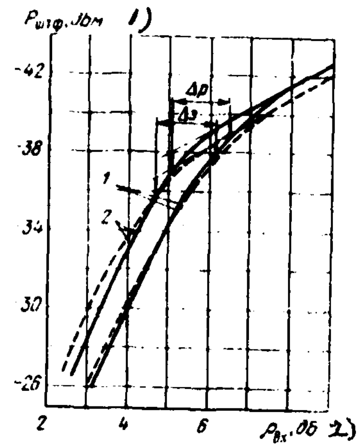

Fig. 4

Key: 1) $R_{shtf,dBm}$; 2) dB

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Analysis and Experimental Study of the Interference Protection of a "Tracking Heterodyne" FM Demodulator with Low Frequency Feedback Circuits

Yu. N. Margolin and E. Ya. Ryskin

The differential equation for the output phase of an FM demodulator was transformed and used to determine the point of emergence of the threshold. It was shown that for the appropriate selection of the device's parameters, the demodulator's interference protection is determined by a band of the SG's [tracking heterodyne] narrow band filter. The results of the study are confirmed experimentally.

Differential equations were derived in [1 and 2] (1) for the phase and the envelope at the output of the device for lowering the threshold level of a "tracking heterodyne" [SG] FM receiver with feedback circuits for a low frequency (OSCh) [3]:

$$\varphi' - \varphi_{г}' = \frac{1}{T}\,\frac{R_{вх}}{V}\sin\left[Q_{вх} - \varphi\right], \qquad (1а)$$

$$V' + \frac{1}{T}\,V = \frac{1}{T}\,R_{вх}\cos\left[Q_{вх} - \varphi\right], \qquad (1б)$$

$$\varphi_{г} = \int\limits_0^t h(\tau)\,\varphi(t-\tau)\,d\tau, \qquad (1в)$$

in which V_{φ} is the envelope and the phase at the output of the second mixer (SM2) (Fig. 1), T is the value of the group delay time created by the single resonance circuit

of the SG filter, φ_r is the phase at the output of the frequency modulating heterodyne (ChMG), $h(\tau)$ is the pulsed reaction of the RC-filter which creates a delay τ_3 in the general circuit of the feedback circuit, R_{BX}, Q_{BX} are the values of the envelope and the phase at the input for the first mixer (SM1). The structural scheme for the SG is shown in Fig. 1.

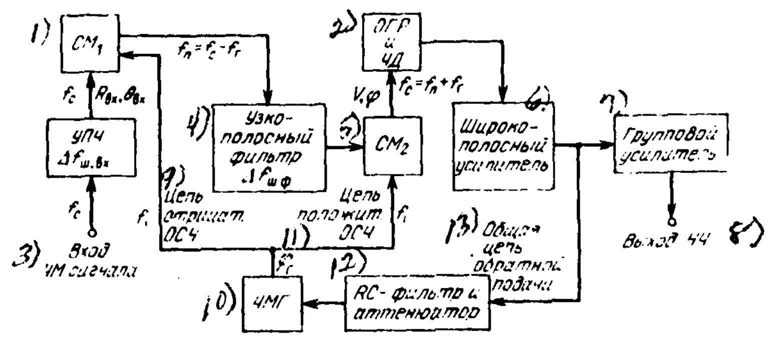


Fig. 1

1) CM1, 2) OGR and ChD, 3) Input of FM signal, 4) Narrow band filter, 5) CM2, 6) Wide band amplifier, 7) Group amplifier, 8) NCh [low frequency] output, 9) Negative circuit OSCh, 10) ChMG, 11) Circuit of positive OSCh [frequency feedback], 12) RC - filter and attenuator, 13) General feedback circuit.

We will present equation (1c) in the operational form $\varphi_r = \varphi/(1+p\tau_3)$ and by means of the usual transformations for (1a) we will arrive at equation (2) for the phase at the output for the second mixer for the presence of an unmodulated carrier at the input to the SG with an amplitude R_c and a noise $\xi_1(t)$ with two-sided spectral density $N_0 = kT_ш$ (where k is the Boltzmann constant and $T_ш$ is the noise temperature in absolute units):

$$\varphi' + \int_0^t \left[\frac{R_c}{T\tau_3 V}\sin\varphi - \frac{\xi_1(t)}{T\tau_3 V}\right] dt +$$
$$+ \frac{1}{T}\,\frac{R_c}{V}\sin\varphi + \frac{1}{T}\,\frac{\xi_1(t)}{V} = 0. \qquad (2)$$

Note that an equation of the type (2) was obtained in [4] for a FAPCh system with a proportional-integrating filter which has a transmitting function F(p) = 1 + a/p. In fact, the following equation is valid for the phase error [4]:

$$\varphi' + a\int_0^t [A\,K\,\sin\varphi + \xi(t)K]\,dt + K[A\,\sin\varphi + \xi(t)] = 0, \qquad (3)$$

in which A is the effective amplitude for the signal at the input to the FAPCh, K is the control coefficient in the loop for the negative feedback of the FAPCh.

In [4], the interference protection of the FAPCh was studied on the basis of the theory for Markov processes assuming that the noise $\xi(t)$ is distributed normally, that δ is correlated and has a one-sided spectral density of N_0. In the same source N has been calculated, i.e., the number of phase jumps for the reference generator for the FAPCh with a proportional-integrating filter

$$N = \gamma/2\pi^2 I_0^2(\alpha), \qquad (4)$$

in which

$$\gamma = N_0\frac{(A\,K + a)^2}{A^2}/4;\ \alpha = 4A^2/N_0(A\,K + a). \qquad (5)$$

It is emphasized, in this, that (4) is an approximate

relationship which has a high accuracy for $a \gg 1$, i.e., when the asymptotic representation

$$I_0(a) \sim e^a / \sqrt{2\pi a}\,. \qquad (6)$$

is valid. In this case

$$N \sim e^{-2a}(AK + a)/\pi. \qquad (7)$$

Later we will use these relationships to calculate the number of jumps and to determine the point of onset for the threshold in an FM demodulator with SG.

Let us turn to equation (2). The number of jumps in a system which is described by this equation can be readily calculated if V is set. In this case we can convert from (2) to (3) if it is assumed that $AK = R_c/TV$, $a = 1/\tau_3$. In so doing, we must keep in mind that the one-sided spectral density for the noise at the input to the SG is equal to $2N_0$. Further, if we follow the argument in [4], we find that for an SG the conditional number of jumps (for a fixed value of V) is determined by the relationship:

$$N_V = \frac{N_0\left(\frac{R_c}{V} + \frac{T}{\tau_3}\right)^2}{4\pi^2 T^2 R_c^2 I_0^2\left[2R_c^2/N_0\left(R_c/TV + 1/\tau_3\right)\right]}\,. \qquad (8)$$

Let us consider equation (1b) which, for $\varphi_r = 0$ is converted into the equation for a single circuit [6], in which the envelope V is distributed according to Rayleigh's generalized law:

$$W(V) = \frac{V}{\sigma_{m}^2} e^{-\left(V^2 + R_c^2\right)/2\sigma_{m}^2} I_0\left(\frac{VR_c}{\sigma_{m}^2}\right). \qquad (9)$$

in which σ_{m}^2 is the power of the noise at the input to the SG's filter. For $\varphi_r \neq 0$, the distribution W(V) is other

than (9), but in the case studied below, which is of practical importance, it approaches (9). The accuracy with which the distribution (9) is fulfilled in a circuit with an SG depends on the τ_3/T ratio. In fact, if $\tau_2/T \gg 1$, then we approach (9) because of the actual opening of the loop of the feedback circuit, however, in this case there will be no compression of the useful signal. If $\tau_2/T \ll 1$ we approach a marked contraction of the useful frequency deviation and the simultaneous deterioration of the circuit's freedom from interference. In this method of analysis, according to (8), this is related to a great increase in N_V even for $V = R_C$. Usually we select $\tau_3/T \approx 1$ in practice. However, in this case, the phase processes at the outputs for the signal circuit filter and the ChMG are poorly correlated and relationship (9) is approximately valid for the envelope's distribution.

If we try to average N_V as determined by expression (8) in conformity with (9), we arrive at a diverging integral. However, if we take into account the fact that the practical SG circuits have, in addition to the single circuit, narrow band fiber, a preselector also (a preliminary UPCh [intermediate frequency amplifier]), then the averaging gives a finite result. The fact is that the expansion of the equivalent noise band for the SG which is related with the expansion of the band for the single-circuit filter (or with the decrease in V) cannot result in a number of jumps for N_V greater than that which is generated by noise in the band for the input UPCh of the demodulator, which allows us to evaluate the lower limit for the integration in terms of V.

We will introduce the designations: $z = V/R_C$, $T/\tau_3=\delta$; $n_*^* = R_C^2T/N_0$ is the signal/noise ratio in the SG's filter

band, $\sigma_{ш}^2 = N_0/2T$ is the power of the noise in the filter band; $k_э = \Delta f_{ш\,вх}/\Delta f_{шф}$ is the ratio of the UPCh's noise band at the input to the SG to the noise band of the SG's filter. We will rewrite (8) in the following form using conventional designations

$$N_V = \frac{\Delta f_{шф}(1+z\delta)^2}{2\pi^2 z^2 n I_0^2 [2n/(1/z+\delta)]}. \qquad (10)$$

If the inequality $\frac{2n}{1/z+\delta} \geqslant 2$ is valid the Bessel function for the imaginary argument $I_0(t)$ is replaced by the asymptotic expression (6) with sufficient accuracy, and then

$$N_V = \frac{4\,\Delta f_{шф} \exp\left[-\frac{4n}{(1/z+\delta)}\right](1+z\delta)}{\pi z}. \qquad (11)$$

As a result of averaging N_V according to (9) we get the following expression to calculate $\bar{N}$

$$\bar{N} = \left(\frac{2\,\Delta f_{шф}\sqrt{n}}{\pi\sqrt{\pi}} \times \right.$$
$$\left. \times \int_{z_0}^{\infty} \frac{\exp\left[-\frac{4nz}{(1+\delta z)} - n(1-z)^2\right](1+z\delta)\,dz}{\sqrt{z}}\right) +$$
$$+ N_{чд} P(z < z_0). \qquad (12)$$

We go about determining z_0 in the following way. The number of jumps $N_{чд}$ at the output of a standard FM demodulator with a rectangular frequency characteristic of the UPCh at the input, equal to $k\Delta f_{шф}$, which is calculated according to [5]:

$$N_{чд} = \frac{k\,\Delta f_{шф}}{2\sqrt{3}} \frac{e^{-n/k}}{\sqrt{\pi n/k}}. \qquad (13)$$

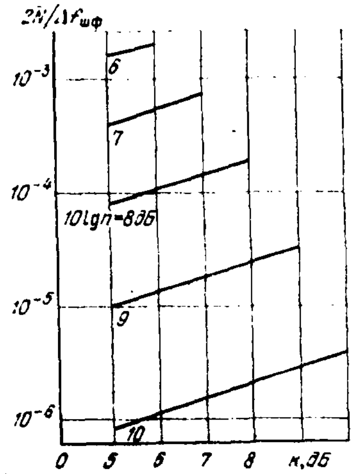


Fig. 2

If we equate the right hand sides of (11) and (13) we come to the transcendental equation for calculating z

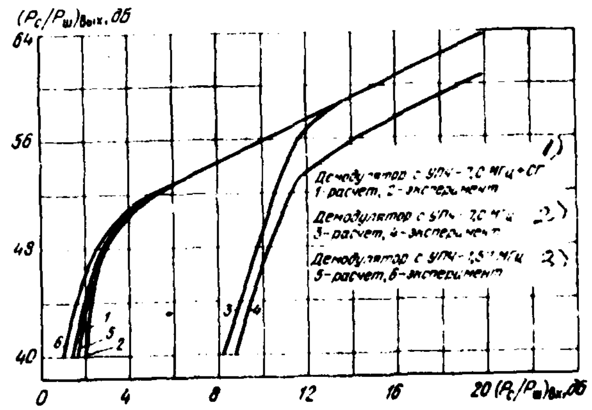
$$\frac{e^{(-4nz_0(1+\delta z_0))}(1+\delta z_0)}{z_0} = \frac{0{,}12\, e^{-n/k}}{\sqrt{n/k}}. \qquad (14)$$

Graphs are shown in Fig. 2 which were calculated by means of equation (12) for different values of n and k and for $\delta = 1$. The values of z_0 are found by means of equation (14).

The expression for the curve of interference protection of an SG is written in the form [5]:

$$\frac{P_{\text{шк}}}{\xi P_c} = \left[\frac{2\bar{N}}{\Delta f_{\text{шф}}} + \left(\frac{\xi F_{\text{к}}}{\Delta f_{\text{шф}}}\right)^2 \frac{1}{n}\right] \frac{\Delta f_{\text{шф}}\, \Delta F_{\text{к}}}{\Delta f_{\text{к}}^2}, \qquad (15)$$

in which Δf_k is the effective deviation in the frequency in the telephone channel: ΔF_k is the telephone channel band which is equal to 3.1 kHz; F_k is the telephone channel carrier.



1) Demodulator with UPCh - 7.0 MHz + SG 1) calculated 2) experimental
2) Demodulator with UPCh - 7.0 MHz 3) calculated, 4) experimental
3) Demodulator with UPCh - 1.57 MHz 5) calculated 6) experimental

Fig. 3

In Fig. 3, Curve 1 shows the interference protection of a demodulator with an SG with the following parameters: noise band for the input UPCh $\Delta f_{\text{ш вх}} = 7$MHz, noise band for the narrow band filter of the SG $\Delta f_{\text{ш}} = 1{,}57$ MHz; $\delta = T/\tau_{з} = 0{,}85$; ($T = 340\mu$s ; $\tau_{з} = 400$ μs). Here the signal/noise ratio in the $\Delta f_{\text{ш вх}}$ band is plotted on the abscissa, the ratio of the signal's power (without predistortion and nonpsophometric) to the noise's power in the telephone channel band ΔF_k = 3.1 kHz with the subcarrier F_k = 14 kHz is plotted along the ordinate. The demodulator with these parameters was built according to the scheme (Fig. 1) and was designed to receive 60-channel telephone communications with an effective deviation of Δf_k = 100 kHz per channel and an effective communications deviation of $\Delta F_{\text{эф}}$ = 280 kHz (load P_{cp} = 9 dB). The deviation in the frequency at the output of the ChMG was made equal to the deviation of the input signal by means of an attenuator in the feedback circuit. In this case, the maximum compression was provided for the signal's spectrum in the narrow band filter of the SG. The experimental threshold dependence for this demodulator in the telephone channel with the subcarrier F_k = 14 kHz is shown in Fig. 3 (curve 2). For comparison the calculated (3) and experimental (4) curves are given in the same figure for a standard FM demodulator with a noise band for the input UPCh $\Delta f_{\text{вх}}$ = 7 MHz.

As we can see from comparing curves 1-4, the gain at the point of onset of the threshold for the demodulator with the SG as compared with a standard FM demodulator is equal to 8.5 dB for a band ratio of $\Delta f_{\text{ш вх}} \cdot \Delta f_{\text{шф}} = 6{,}5$ dB. It is important to note that at the point having a zero measuring level which is psophometric with predistortions, the power of the transmitted noise which is introduced by the SG into the channel of the demodulator and which is

measured in the upper telephone channel with the subcarrier F_k = 256 kHz for Δf_k = 100 kHz and P_{cp} = 9 dB did not exceed 1000 pW. In this case, the shift in the point of onset for the threshold for curve 2 did not exceed 1 dB.

For this demodulator with an SG for $\Delta f_{ш\,вх}$ = 7 MHz, a calculation was made of the relation of the number of jumps to the value of $\tau_з$. Thus, for $\tau_з = 0{,}7\ T$ there was a slight increase in the number of jumps as compared with the case of τ = T which resulted in a change in the signal/noise ratio at the output of the demodulator (in the telephone channel) of no more than 2 dB.

There are two other curves given in Fig. 3 which are of interest. Curve 5, which was calculated by the method given in [5] for the interference protection for an FM demodulator with an input band filter which has a noise band $\Delta f_{вх}$ = 1.57 MHz, i.e., equal to the band for the narrow band filter of the SG and the experimental curve (6) which corresponds to it.

It follows from a comparison of Curves 1 and 2 for the SG and 5 and 6 for the UPCh with $\Delta f_{ш}$ = 1.57 MHz that they are quite similar.

The dependence of the signal/noise ratio was also checked experimentally in the channel at the output of the demodulator with the SG as a function of the input band $\Delta f_{ш\,вх}$ for a fixed width of the noise band for the SG filter equal to 1.57 MHz. The threshold dependences were measured for an FM demodulator with an SG for the preselector bands 4.9, 12.7 MHz$\left(\Delta f_{ш\,в}/\Delta f_{ш\,ф} = 9.1\text{ dB}\right)$. For a broader band at the input the point of onset of the threshold is shifted by approximately 1 dB and the signal/noise ratio in the channel (in the subthreshold region) shifts

by 5 dB. We can see from the graphs (Fig. 2), the results of the calculation are in good agreement with these experimental data.

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Tracking Process in Demodulators for FM Signals with Frequency Feedback

Yu. V. Savinov

An approximate method is given for calculating the discrimination characteristics of a demodulator for FM signals with frequency feedback. The effect of amplitude detection in the ChD on the tracking process and on the threshold mechanism was studied.

The tracking properties and the threshold mechanism for FM signal demodulators with frequency feedback (OSCh), as shown earlier [1], depends greatly on their discrimination characteristics. The analysis given in [1] was based on the representation of the discrimination characteristics as the characteristics for an ideal ChD with a linear segment limited because of the attenuation of the signal on the filter slopes for the intermediate frequency (FPCh) in the feedback circuit. However, such characteristics, although they reflect some of the features of the threshold mechanism, differ from the experimental characteristics.

In this work the form of the discrimination characteristics is refined for demodulators with OSCh as a function of the FPCh band, the ChD's parameters, and the carrier/noise ratio. The results are used to study the effect of the amplitude detection in the ChD on the process for tracking scatterings for the instantaneous frequency for the sum of the signal and noise at the demodulator's input.

Discrimination characteristics of demodulators with OSCh

It follows from the structural scheme of the demodulator shown in Fig. 1 that the static discrimination characteristics

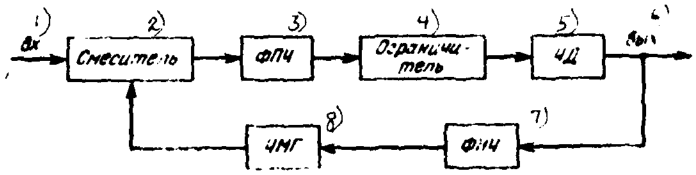

Fig. 1

1) Input, 2) Mixer, 3) FPCh, 4) Limiter, 5) ChD, 6) Output, 7) FNCh, 8) ChMG

are formed by the FPCh, the limiter, and the ChD. In practice, ChD's with unbalanced circuits are usually used and the FPCh is made in the form of a single circuit. Let the following signal act at the input of this channel

$$U_0 = U\cos[2\pi(f_0 + \Delta f) + \varphi_0], \qquad (1)$$

in which f_0 is the nominal, intermediate frequency and the transmitting frequency of the ChD, Δf is the deviation of the intermediate frequency from the nominal value. Then, the generalized discrimination characteristic, without taking into account the action of the limiter, can be represented as:

$$\psi(a) = \frac{1}{\sqrt{1+(ab)^2}}\left[\frac{1}{\sqrt{1+(a_0-a)^2}} - \frac{1}{\sqrt{1+(a_0+a)^2}}\right], \qquad (2)$$

where $$a = \frac{2\Delta f}{f_0}Q_d; \quad a_0 = \frac{2\Delta f_d}{f_0}Q_d;$$

Δf_d, Q_d are the frequency difference and the quality of the ChD circuits, Q is the quality of the FPCh, $b = Q/Q_d$. This graphis for $\psi(a)$, calculated by means of expression (2) for b = 10 are shown in Fig. 2. It follows from the

graphs that the boundaries for the linear segment for the discrimination characteristics does not exceed the FPCh's transmission band. As the generalized frequency difference a_0 is increased the slopes for $\psi(a)$ gradually are rectified. The maximum curvature for $\psi(a)$ is reached for $a_0 = 1/\sqrt{2}$. The slopes for the discrimination characteristic of the ChD do not have a significant effect on the form of $\psi(a)$. This is explained by the fact that a much wider FPCh band is chosen in order to lower the delay time for the signals in the ChD's transmission band. When there is an initial frequency difference between the FPCh's resonance frequency and the ChD's transmission frequency the characteristics for $\psi(a)$ become asymmetric.

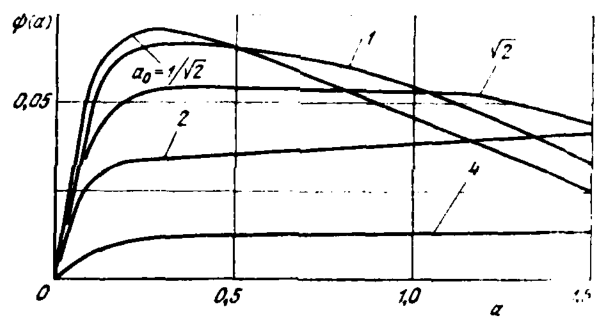

Fig. 2

A further refinement of the discrimination characteristic is associated with taking into account transition processes as well as the effect of the limiter and the signal/noise ratio. In the general case the limiter causes some expansion of the linear segment which can be analyzed by means of expression (2). Let us consider the action of the signal and the additive fluctuation noise on the ChD by using the method described in [2]. In the general form we get

$$\psi(a) = \frac{1}{\sqrt{1+(ab)^2}}\left[\frac{B(\rho_1)}{\sqrt{1+(a_0-a)^2}} - \frac{B(\rho_2)}{\sqrt{1+(a_0+a)^2}}\right], \tag{3}$$

where $B(\rho_1)$ and $B(\rho_2)$ are factors which take into account the amplitude detection in the ChD

$$\rho_1 = \rho\{[1+(ab)^2][1+(a_0-a)^2]\}^{-1}; \tag{4}$$
$$\rho_2 = \rho\{[1+(ab)^2][1+(a_0+a)^2]\}^{-1} \tag{5}$$

are the carrier/noise ratio at the input for the amplitude detectors, ρ is the carrier/noise ratio in the FPCh for $\Delta f = 0$. We will consider the case in which the increase in the constant component at the output to the ChD is the rectified signal in the feedback circuit and inertialess, single-half period detectors with a ν-power characteristic are used as the amplitude detectors. It follows from the analysis of the conversion of the normal noise and signal (1) in the nonlinear elements, which was given in [3], that for $\nu = 1$

$$B(\rho_1) = \frac{\pi}{2\sqrt{\rho_1}} e^{-\rho_1}\left[(1+\rho_1) I_0\left(\frac{\rho_1}{2}\right) + \rho_1 I_1\left(\frac{\rho_1}{2}\right)\right], \tag{6}$$

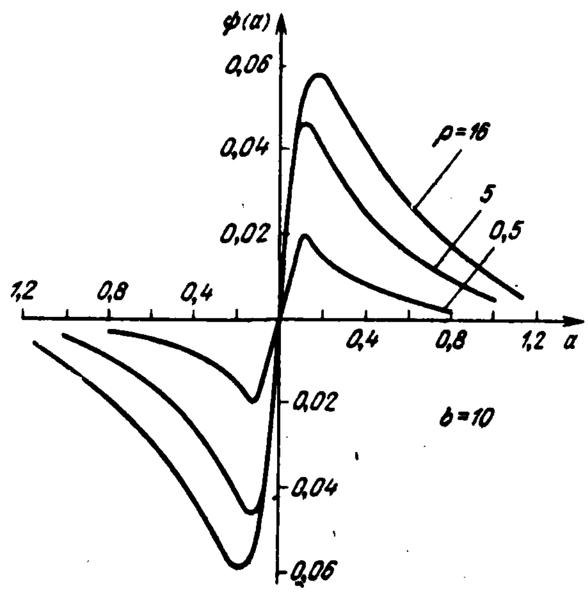

Fig. 3

in which $I_0(z)$ and $I_i(z)$ are Bessel functions for the imaginary argument of the zero and first orders.

The expression for $B(\rho_2)$ has an analogous form. The generalized discrimination characteristics, calculated for different values of ρ and $a_0 = 1/\sqrt{2}$ by means of expressions (3) - (6) are shown in Fig. 3, from which we can see that there is a marked decrease in the linear segment for the discriminator beginning at $\rho < 5$.

Dynamic characteristics. The dynamic characteristics show the relationship between the deviation for the signal's frequency at the input to the demodulator Δf_m and the deviation for the signal's frequency at the output of the mixer Δf in the feedback loop. If the characteristics of the frequency-modulated heterodyne (ChMG) and the discriminator are known the graphic method [2], which was used to design the APCh system, can be used to find the dynamic characteristics under the established conditions.

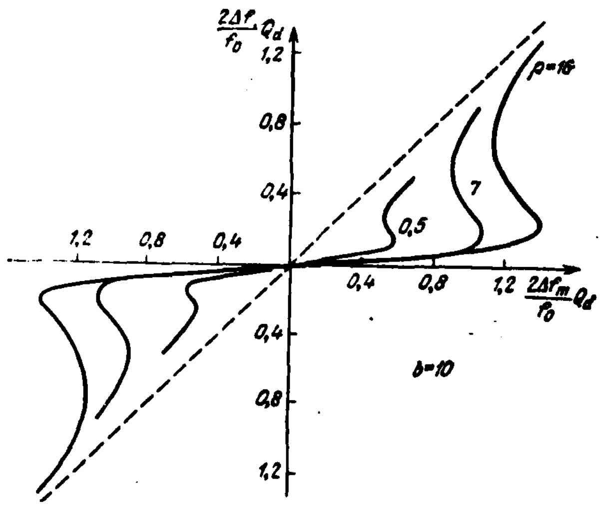

Fig. 4

The generalized dynamic characteristics are shown in Fig. 4 which were set up for the assumption that the modulation characteristic of the ChMG is linear, its curvature is $S_g = 20$ and $a_0 - 1/\sqrt{2}$. The important parameters of the demodulator with OSCh can be determined from the dynamic characteristics: the capture band B_m and the holding band B_h and their dependence on ρ. The value of B_h in the suprathreshold region and near the threshold must exceed the frequency deviation Δf_m with some reserve which takes into account the fluctuation in the frequency of the sum of the signal and noise at the input to the demodulator. On the other hand, an increase in B_h is equivalent to expanding the effective noise band of the demodulator B which, as we know, decreases its freedom from interference. Probably the most practical solution in choosing B_h comes down to the fact that the operating segment of the dynamic characteristic near the threshold for improving the FM should be taken as equal to B_m and

$$B_m = 2\Delta f_m. \tag{7}$$

It should be pointed out that the noise which passes along the feedback circuit causes random detuning of the ChMG and, as a result, unstable zones appear in the vicinity of the boundaries for B_m and B_h. The width of these zones increases with a decrease in the signal/noise ratio at the input of the demodulator. An analogous phenomenon was noticed in APCh systems in [2].

Remarks on the threshold mechanism. Let us present the sum of the FM signal s(t) and the normal noise n(t) at the demodulator's output in the following way

$$\begin{aligned} v(t) = A\cos[\omega t + \psi(t)] + N(t)\cos[\omega t + \theta(t)] = \\ = r(t)\cos[\omega t + \varphi]. \end{aligned} \tag{8}$$

in which ψ (t) is the phase for s(t), N(t) and θ(t) are the envelope and the phase for n(t), respectively.

$$r(t) = \sqrt{A_2 + N^2 + 2AN \cos(\theta - \psi)};$$
$$\varphi = \psi + \operatorname{arctg} \frac{N \sin(\theta - \psi)}{A + N \cos(\theta - \psi)}.$$

The physical picture of the threshold phenomena in demodulators with OSCh for the action of a signal v(t) has been studied many times [1, 4-9, etc]. In [4] it was shown experimentally that for a demodulated carrier $\psi(t) = 0$ and deep feedback the threshold pulses appear at the demodulator's output when the carrier/noise ratio ρ in the band B is approximately equal to 6.8 dB. It was shown in [7] that the feedback threshold arises as the result of tracking the phase jumps φ by $\pm 2\pi$. The modulation of the carrier can lead to the regeneration of the threshold pulses directly in the demodulator [1, 6, 9]. It was suggested in [9] that this phenomenon occurs when scatterings having different signs of the instantaneous frequency (doublet) occur at the demodulator's input for the signal v(t) and through r(t) causing a narrowing of the holding band.

The physical concept of the threshold mechanism for demodulators with OSCh can be expanded if we consider that the narrowing of the holding band B_h, as shown above, depends on the amplitude detection of the signal and noise sum in the ChD and the associated suppression of the signal by the noise. It follows from (6) that the troughs for the envelope r(t) and the corresponding change in the instantaneous signal/noise ratio at the input for the amplitude detectors of the ChD cause a marked narrowing of B_h as compared with its value in the suprathreshold region. It should be pointed out that the limiter in

the feedback circuit cannot eliminate deep troughs r(t) since for a weak signal its suppression by noise even occurs in the limiter.

Let us consider the case in which the troughs r(t) are accompanied by scatterings in the instantaneous frequency ω of the signal v(t) of the "doublet" type. If the area of the scatterings ω_m, S, above one of the boundaries of B_h reaches some critical value S_{kp}, then the feedback circuit, as usually happens in tracking systems, is interrupted. In the process for the reverse entry into synchronism, at the moment the capture band's boundaries reach ω_m threshold pulses can arise. This phenomenon is characteristic for FM signals since for an unmodulated carrier it is most probable that its instantaneous frequency after the trough r(t) is finished, will be in the capture band.

When the troughs r(t) are accompanied by jumps in φ by $\pm 2\pi$ the tracking of the frequency scatterings is possible under the condition that their amplitude $\varphi_p' < B_h/2$. Moreover, the duration of the scatterings must exceed the time for the delay of the signals in the feedback circuit. Since these conditions are not usually fulfilled in the suprathreshold and near threshold regions the scatterings are suppressed. For an unmodulated carrier and a carrier/noise ratio of $\rho_0 <$ 6 dB, i.e., in the threshold region and below, the value of φ_p' in a filter with a rectangular frequency characteristic and a band B_0 which is equal approximately to twice the inertia radius of the sum [6]. Thus, if we assume that the frequency characteristic of the closed feedback circuit is rectangular and $B_0 = B_h$, then $\varphi_p' \approx B_h/\sqrt{3}$, i.e., the tracking of the scatterings begins approximately at $\rho_c \leqslant$ 6 dB.

The tracking of frequency scatterings for a modulated carrier depends on the type of modulating oscillation and the frequency deviation at the moment of a jump in φ by $\pm 2\pi$. If the modulating oscillation is sinusoidal and $\Delta f_m = B_0/2$ then, for $\rho_0 = 6$ dB, the most probable value is $|\varphi_p^{\cdot}| = 3B_0$ [10], i. e. $|\varphi_p^{\cdot}| = 3B_h$.

Consequently, the tracking of frequency scatterings is not very probable. When the modulating oscillation is more complex, for example, a speech signal, the number of threshold pulses tracked and created by the demodulator changes continuously. The greatest lowering of the threshold will be achieved for signals whose instantaneous frequency exceeds the capture band for the smallest period of time. As in the preceding case, the interruption of the circuit is possible if $S > S_{kp}$. In this case, the suppression of the jumps or their appearance when the demodulator enters into synchronism depends on the position of the instantaneous frequency ω_m with respect to the boundaries of the capture band at the mement the trough r(t) is complete. Here, apparently there is an analogy with the tracking process and the origin of jumps in the FAPCh systems [11].

In conclusion let us note that the phenomenon of signal suppression by noise in amplitude detectors of ChD greatly lowers the demodulator's freedom from interference in the subthreshold region.

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Experimental Study of the Interference Protection of Tracking Demodulators of FM Signals in the Subthreshold Region

Yu. S. Agapov, V.M. Dorofeyev,
and V.L. Platonov

The results are given on an experimental study of the interference protection of a synchrono-phase demodulator (SFD) for FM signals.

It is shown that as the signal/noise ratio becomes worse the gain in the tracking reception decreases in comparison with the frequency detector (ChD) and the freedom from interference for the ChD and the SFD are almost identical in the weak signal range.

In modern FM signal reception on a background of high level noise, methods of tracking reception are being used more and more extensively. However, in contrast to the standard ChD, a precise analysis is unknown today of the freedom from interference for tracking demodulators of FM signals in the subthreshold region. Approximate methods of analysis are based on the calculation of the average number of breaks in the synchronism due to the action of noise. Along with a partial accounting for the periodic nonlinearity, as a rule, a linear approximation is used to calculate the dispersion in the phase error [1]. Therefore the available results, strictly speaking, remain valid for the initial subthreshold region for which the linear model for reception holds with a high probability.

It should be pointed out that in the known methods the analysis of the spectral density of the noise at the

demodulator's output is characterized by an average value for the number of jumps by 2π for the phase difference for the oscillations for the supporting ChM of the heterodyne and the signal. This determination is only possible for a Poisson flow for the jumps.

If we refuse to use the linear approximation and, even more so, if we do not try to take into account the statistical relationship between the moments for the appearance of phase jumps for the oscillations of the FM heterodyne, we encounter significant mathematical difficulties. In addition, today there are no methods for calculating the suppression of the frequency modulation by noise in the case of tracking reception. All of this emphasizes the importance of the experimental study of the interference protection for tracking reception in the subthreshold region which is now the only possible method of investigation in the case of a rather weak signal.

In this article the results are given of measuring the interference protection of a synchrono-phase detector and a standard detector with a limiter for the frequency-modulated signals, which are used to transmit in multichannel telephony, which is close to ideal.

The measurements were made using FM demodulators designed to receive 60-channel telephone communications with an effective deviation of 400 kHz per channel.

In the experiment SFD were used with a proportional-integrating filter and a standard FM detector with a PCh filter having a variable transmission band at the input.

The experimental threshold curves, which are given in Fig. 1 show the dependence of the signal/noise ratio at the input in the AM band (ρ_{AM}) and the signal/noise ratio at the output to the telephone channel ($P_c/P_ш$), and the suppression coefficient (β) also.

If there is predistortion of the spectrum for the group signal in the lower telephone channel in the subthreshold region the signal/noise ratio will be the worst. On this basis the measurements were made for the lower channel with an average frequency of 14 kHz. Curve 1 shows the signal/noise ratio in the channel and Curve 2 shows the suppression of the signal for SFD.

It is essential to indicate that the threshold curves 3, 4, 5, and 6 for ChD were taken for values of the ratio of the transmission band width for the PCh filter to the mean square deviation for multichannel communication, equal to 7.5, 6, 5, and 4 respectively which, as was shown in [2], affects the freedom from interference greatly. In constructing the threshold curves the psophometric effect

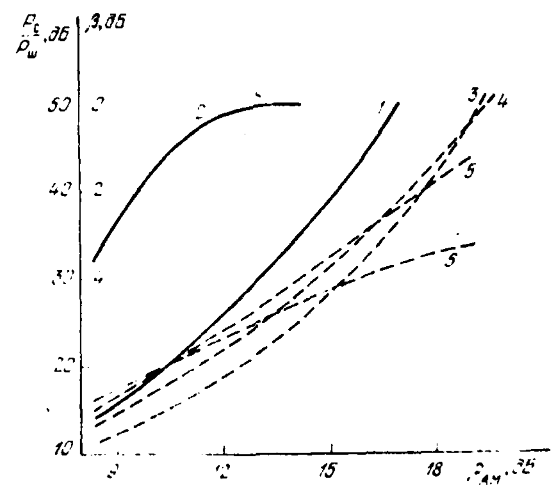

Fig. 1

was taken into account as well as the decrease in the signals deviation by 4 dB in the lower channel for the presence of the standard predistortions.

We can see from a comparison of the threshold curves that the SFD loses gain as the signal/noise ratio drops and, for a weak signal, the ChD loses in terms of the freedom from interference. During the measurements it was noted that the use of a PCh filter with a variable transmission band in front of the SFD does not improve its freedom from interference markedly.

The exception is the case of a weak signal. In this case it is possible to get the same results as for a ChD which is generally apparent. On the other hand, if a ChD is used there is a clear-cut optimum in terms of the PCh filters transmission band [2]. The value of the optimum transmission band decreases markedly for the decrease in the signal/noise ratio and the freedom from interference for an FM receiver with a variable transmission band increases somewhat in the subthreshold region.

This, specifically, explains the decrease in the effectiveness in using tracking reception. However, even for a constant transmission band for the PCh filter the threshold curve for the SFD for a weak signal has a sharper drop (Curves 1 and 3 in Fig. 1) and the suppression of the wanted modulation increases more rapidly than that which follows from the known relationship for the ChD $\beta = (1 - e^{-\rho})^2$, where ρ is the signal/noise ratio in the FM band.

The features in the behavior of the SFD's characteristics in the threshold region, which were studied above, are

determined by the same principle for tracking reception in which the phase of the signal is evaluated and a copy of the signal is formed by means of the voltage of the frequency-modulated heterodyne to develop additional filtering of the signal from the noise. For a large signal/noise ratio the copy reflects the signal quite well and the ChMG's phase is determined mainly by the phase of the signal and it depends only slightly on the realization of the noise at the input to the demodulator. This assures good efficiency for the additional filtering. In the range for a weak signal the power of the noise at the output of the demodulator becomes commensurate with the power of the demodulated signal. In this case, as the signal/noise ratio decreases the dispersion of the phase of the ChMG increases as does the dependence of the ChMG's phase on the realization of the input noise. As a result the tracking of the noise sets in, with a high degree of probability and the effectiveness of the additional filtering decreases. The latter results in a greater deterioration in the signal/noise ratio at the output of the tracking demodulator.

In conclusion, let us point out that for a sufficiently weak signal the tracking reception is almost equivalent, in its freedom from interference, to reception using a standard ChD if the PCh filter is optimal.

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Effect of the Statistical Dependence of the Phase Jumps by 2π in the Sum of the Signal and the Narrow Band Noise on the Intensity of the Frequency Fluctuations

Yu. S. Agapov

The effect was demonstrated of the statistical dependence of the phase jumps on the frequency fluctuations. An expression is given for the spectral density of the fluctuations at a zero frequency for a frequency detuned signal. The results are compared with the results calculated using Rice's method.

A knowledge of the spectral density for the fluctuations of the derivative phase at zero frequency $F_{\varphi'}(0)$ for the sum of a VCh signal and a narrow band noise can be used in many practical cases to determine the freedom from interference for FM signal demodulators. Today the pulse model (Rice) for the interaction of the signal with noise [1] is widely and effectively used to calculate $F_{\varphi'}(0)$. However, the relationships obtained by O.S. Rice are not sufficiently accurate for a low signal/noise ratio. The results for this region were refined in [2] by taking into account the statistical relationship between the moments at which jumps in the phases by 2π appear. The effect of the statistical dependence is also reflected by the model which was suggested by N.M. Blachman [3] in which the zeros in the VCh process were studied. Both models give values for $F_{\varphi'}(0)$ which are valid for an arbitrary signal/noise ratio. However, the authors of [2 and 3] studied a nonmodulated signal tuned to the central frequency of a PCh filter. It is shown below how the statistical

dependence of the phase jumps affect the values of $F_{\varphi}(0)$ for the frequency detuning of the signal. The results are obtained by means of some transformation of the zero model suggested by Blachman in [3].

We will make some generalization of the known models for the interaction between a signal and noise. For this we will consider the method for taking into account the statistical dependence of the phase jumps by 2π.

The sum of the signal and the narrow band noise is written in the form

$$\eta(t) = E(t) \cos[\omega_0 t + \varphi(t)],$$

where E(t) and $\varphi(t)$ are the slowly changing amplitude and phase, ω_0 is the mean frequency of the energy spectrum for the noise.

The spectrum for the noise is assumed to be symmetrical with respect to ω_0.

Let us consider the intersection of the phase $\varphi(t)$ of equidistant levels with a step Δ radians in the time period (-T/2, +T/2). The vector diagram for the process η is hown in Fig. 1. The phase for this process passes consecutively through the $0, \pi/2, \pi$ levels 0, $\pi/2$, π, $-\pi/2$, and again π but in the reverse direction.

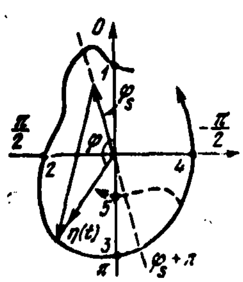

Fig. 1

The step between such levels is $\Delta = \pi/4$.

We will determine N_T as the difference between the number of intersections for a positive value of the derivative phase $\psi'(t)$ and the number of intersections for its negative value (in Fig. 1 the intersections of the first type are designated on the hodograph of the vector η by the points 1, 2, 3, and 4, and by point 5 for the second type.

This difference is determined, with an accuracy to unity, by the increase in the phase φ in the time T divided by the value of the step between the levels, i.e.,

$$N_T = \frac{1}{\Delta} \int_{-T/2}^{T/2} \varphi'(t)\, dt. \qquad (1)$$

By calculating the ratio of the second moment from the left and the right hand side of expression (1) to the time period T and converting to the limit for $T \to \infty$, we can show that the spectral density of the derivative phase ψ' at zero frequency is

$$F_{\varphi'}(0) = \lim_{T \to \infty} \frac{2\Delta^2}{T} \langle (N_T)^2 \rangle . \qquad (2)$$

Here $\langle\ \rangle$ indicates averaged over great numbers.

An analogous method was used in [3] in which the zero process $E(t) \cos \psi(t)$ was studied, which is equivalent to intersections of the phase φ with the $\pm\pi/2$ levels.

Without destroying the general nature of the discussion, the calculations for N_T can be restricted to the region in which φ has a unique value for the vector representation of the process $\eta(t)$. It will also be assumed that

$$\Delta = 2\pi/k, \qquad (3)$$

where k is an arbitrary whole number.

Further, we show that if the derivative phase contains a constant component then the average value is not equal to zero only in this case and the discrete component of the spectrum corresponds to it for a zero frequency of the spectrum for the derivative phase.

The zero component for the continuous part of the spectrum for the derivative phase $F_{\Delta\varphi'}$ (0) is determined by the dispersion in the difference for the number of intersections $\sigma^2_{N_T}$. From this, taking (2) and (3) into consideration, we can write

$$F_{\Delta\varphi'}(0)=\lim_{T\to\infty}\frac{2\pi^2}{Tk^2}\sigma^2_{N_T}. \tag{4}$$

We will show that the given method for determining $F_{\varphi'}$ (0) encompasses the known models for the reaction of a signal with noise.

Let us assume that the moments of time of intersection with the phase of each individual level are statistically independent of one another. In this case the dispersion in the difference of the intersections of each nth level σ^2_{nT} will be equal to the mean value of the total number of intersections M_{nT} for this level for positive and negative values for the derivative phase because of the well known property for the Poisson distribution

$$\sigma^2_{nT}=\langle M_{nT}\rangle,\quad n=1,\ 2,\ 3,\ \ldots,\ k.$$

In addition, it follows from (4), which is valid for any values of k, that $\sigma_{NT} = k^2\sigma^2_{nT}0$, i.e., the fact of the intersection of one of the levels with the phase with

a probability equal to unity, fixes the intersection of all of the remaining k - 1 levels with it (in Fig. 1 the hodograph of the vector η is shown by a solid line). In fact, in the opposite case, the phase must intersect the given level in the opposite direction (in Fig. 1 the hodograph for the vector η is shown by the dotted line). However, the reverse intersection for the level, being a consequence of the direct intersection, must depend on it, which is a very acceptable assumption.

As a result we obtain a group of independent 2π phase jumps which characterize the pulsed component of the noise in the Rice model. On the other hand, it is apparent that the moments of time for the intersection of the phase of any level for the group of independent 2π phase jumps are statistically independent.

As a result we have, according to (4), the following expression for $F_{\varphi'}(0)$

$$F_{\varphi'}(0) = 8\pi^2 \langle M_n \rangle,$$

in which $\langle M_n \rangle$ is the average number of intersections for the nth level in a unit of time.

It is obvious that the condition that the intersections are independent can be realized with a high degree of probability for a good signal/noise ratio if we assume k = 1 and if the value for the level is taken as equal to $\varphi_s + \pi$, where φ_s is the signal's phase. Thus, we finally arrive at the model proposed by Rice [1].

We note that in the case of detuning or modulation of the model with respect to frequency in the Rice model,

the level whose intersection is set must change with an angular velocity equal to the detuning of the signal with respect to ω_0 which introduces a large additional error for a small value of the signal/noise ratio in calculations using the Rice formula. In fact, as the signal's power drops to zero the average number of intersections for this level depends on the detuning of the signal (a "memory" remains of the signal) whereas it is obvious that the effect of modulation of the signal for frequency fluctuations should disappear as the signal/noise ratio decreases.

Let us turn to a study of the dependent intersections. As was shown above, the number of levels in the region of a unique value and their values may be arbitrary. It follows from this that $F_{\varphi'}(0)$ in this case too will be determined by the jumps, or more precisely, by the increases in the phases by 2π but taking into account the statistical dependence between the moments of their appearance. We note, however, that the proper selection of the intersection levels in the initial model can simplify the mathematical calculations and the final expressions for $F_{\varphi'}(0)$ which give the same results upon calculation.

A model was studied in [2] in which the intersections of the phase φ was fixed with levels equal to π for $\Delta=2\pi$. The moment for the intersection of this level and the sign for φ' were determined by means of the product of the delta function and a power function, respectively, of the orthogonal and opposite - phase components of the overall process with respect to the signal. This allowed us to take into acocunt the statistical dependence of the moments of time of the intersections. However, the final results were obtained in rather cumbersome form. In [3] the zero process $\eta(t)$ was studied which, as was

shown above, is equivalent to studying the intersections of the levels $\pm 2\pi$ with the phase φ for $\Delta=\pi$.

In this case, the value of N_T is expressed in terms of the Stilts integral [3]:

$$N_T = -\frac{1}{2}\int_{-T/2}^{T/2} \operatorname{sgn} Y\, d\operatorname{sgn} X, \tag{5}$$

where

$$Y = E(t)\sin\varphi(t), \quad X = E(t)\cos\varphi(t).$$

The spectral density of the derivative phase at a zero frequency in this case, according to [3] will be equal to

$$F_{\varphi'}(0) = \lim_{T\to\infty}\frac{\pi^2}{2T}\left< \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} \operatorname{sgn} Y_1 \operatorname{sgn} Y_2 \times \right.$$

$$\left. \times \frac{d^2(\operatorname{sgn} X_1 \operatorname{sgn} X_2)}{dt_1\, dt_2}\, dt_1\, dt_2 \right>, \tag{6}$$

in which $X_1 = X(t_1),\ Y_1 = Y(t_1),\ X_2 = X(t_2),\ Y_2 = Y(t_2)$.

Some modification of the last model can greatly simplify the mathematical transformations and, as a result, new results are obtained.

Let us write equation (6) for $F_{\varphi'}(o)$ in a somewhat different way. To do this, we will integrate it by parts with respect to the variable t_1. As a result, we get

$$F_{\varphi'}(0) = -\lim_{T\to\infty}\frac{\pi^2}{2T}\left< \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} \operatorname{sgn} X_1 \frac{d\operatorname{sgn} Y_1}{dt_1} \times \right.$$

$$\left. \times \operatorname{sg} Y_2 \frac{d\operatorname{sgn} X_2}{dt_2}\, dt_1\, dt_2 \right>. \tag{7}$$

Here $F_{\varphi}'(0)$ is determined by the limit for the difference in the number of intersections for the levels 0 and π and the levels $\pm\pi/2$, with respect to the mixed second moment.

As an example, let us consider the case of an unmodulated signal with detuning with respect to the mean frequency of the energy spectrum of the noise $\omega = \omega_s - \omega_0$, where ω_s is the signal's frequency. We will assume that the noise has a normal distribution with the parameters $(0, \sigma)$ in this case

$$X(t) = A_c(t) + A_m \cos\Delta\omega t,\ \ Y(t) = A_s + A_m \sin\Delta\omega t,$$

where A_m is the signal's amplitude and A_c and A_s are the quadratic components of the narrow band noise.

Since the chance processes $A_c(t)$ and $A_s(t)$ are mutually independent than we can write (7) in the form

$$F_{\varphi'}(0) = -\lim_{T\to\infty}\frac{\pi^2}{2T}\int_{-T/2}^{T/2}\int_{-T/2}^{T/2}\frac{\partial<\operatorname{sgn} Y_1 \operatorname{sgn} Y_2>}{\partial t_1}\times$$
$$\times\frac{\partial<\operatorname{sgn} X_1 \operatorname{sgn} X_2>}{\partial t_2}\,dt_1\,dt_2. \qquad (8)$$

The expression for the integrand in (8) is the product of the derivatives of the correlation functions for the chance processes after nonlinear sign transformation of the normal chance processes Y(t) and X(t). Both factors depend on the time only through the correlation coefficient $R(t_2 - t_1)$ and the mean values $b_1 = A_m \sin\Delta\omega t_1$ and $a_2 = A_m \cos\Delta\omega t_2$ for the processes Y(t) and X(t), respectively. This means that the so-called method of derivative calculation

of the correlation function can be used effectively to determine $F_{\varphi'}(0)$.

The rule for the differentiation of a complex function of two variables [4] can be applied to each of the factors in (8). By using this method it is simple to find the partial derivatives of the sgn function with respect to R, b_1, and a_2. Further calculations are rather cumbersome. The final result of the transformation (8) can be given in the form of the sum $F_{\varphi'}(0) = F_{\Delta\varphi'}(0) + F_{\delta_{\varphi'}}(0)$, in which $F_{\delta_{\varphi'}}(0)$ is the discrete component of the spectrum φ'. The continuous part of $F_{\varphi'}(0)$ will be

$$F_{\Delta\varphi'}(0) = 2e^{-\rho} \int_{-\infty}^{\infty} \frac{R'^2}{1-R^2} e^{-\rho \frac{1+R^2-2R\cos\Delta\omega\tau}{1-R^2}} d\tau +$$
$$+ 4\Delta\omega e^{-\rho} \int_{-\infty}^{\infty} \frac{-R'\sin\Delta\omega\tau}{1+R^2-2R\cos\Delta\omega\tau} \times$$
$$\times \left(1-e^{-\rho\frac{1+R^2-2R\cos\Delta\omega\tau}{1-R^2}}\right) dt +$$
$$+2\Delta\omega^2 e^{-\rho} \int_{-\infty}^{\infty} \left[1 - e^{-\rho} - \frac{1-R^2}{1+R^2-2R\cos\Delta\omega\tau} \times \right.$$
$$\left. \times \left(1-e^{-\rho\frac{1+R^2-2R\cos\Delta\omega\tau}{1-R^2}}\right)\right] d\tau, \qquad (9)$$

in which $\tau = t_2 - t_1$.

A more detailed derivation of equation (9) is given in [5].

The result found in [3] for $F_{\varphi'}(0)$ is a partial case of (9) for $\Delta\omega = 0$. In this case

$$F_{\varphi'}(0) = 2e^{-\rho} \int_{-\infty}^{\infty} \frac{R'^2}{1-R^2} e^{-\rho\frac{1-R}{1+R}} d\tau.$$

Calculations were made using equation (9) for the filter's Gaussian characteristic, i.e., for $R(\tau)=e^{-\tau^2 r^2/2}$, in which r is the radius of inertia for the filter. The results of the calculaton which are the dependence of the normal spectral density $F(0)/\Pi$ on ρ for different values of the normalized detuning $\gamma=\Delta\omega/2\pi r$, are shown in Fig. 2. Here $\Pi=\sqrt{2\pi}r$ is the noise band of the filter, $F(0) = F_{\dot{\varphi}}(0)/4\pi^2$. For comparison, the results of calculations of the spectral density of the frequency by the Rice method [1] are shown in Fig. 2 by the dotted lines. The dependence of $F(0)e^{\rho}/\Pi$ on γ is shown in Fig. 3 for several values of ρ. As we can see from the graphs, the spectral density F(0) for a large signal/noise ratio coincides with Rice's results which was to be expected since the statistical relationship between the time of the phase jumps by 2π is slight in this case. Further, for low signal/noise ratios in which the statistical relationship for the jumps is large, Rice's formula gives a large error in the calculation, particularly for large frequency differences. The latter is associated, to a large extent, with the "memory" effect mentioned above for the signal in the Rice model. We will also point out that F(0) depends greatly on the detuning for $\rho \gtrsim 1$. For $\rho < 1$, in contrast to Rice's results, the effect of detuning decreases with a decrease in ρ and gradually disappears for small enough values of ρ. A comparison of the curves in Fig. 2 shows that the statistical dependence for the phase jumps always decreases the spectral density of the fluctuations for its derivative at zero frequency which is natural since it is determined by the dispersion in the change values with a mean value of zero.

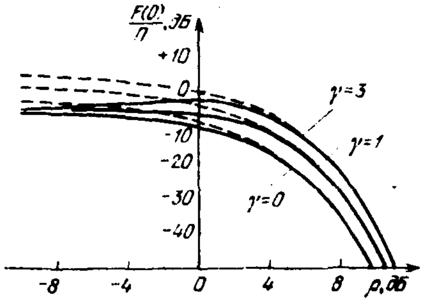

Fig. 2

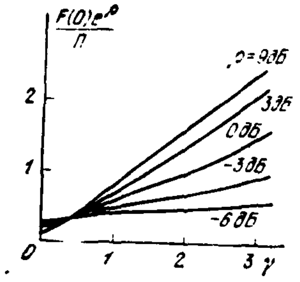

Fig. 3

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Action of Interfering Signals on a Synchrono-Phase Demodulator and the Selectivity Requirements for a Tracking Receiver for FM Signals

V.V. Loginov

The action was studied of useful and interfering FM signals in a synchrono-phase demodulator (SFD). Formulas are found for calculating the permissible, protection ratio of the interfering signal/useful signal ratio in a tracking receiver for telephone channels and also for the signals in multichannel telephony. The selectivity requirements were determined for a tracking receiver.

The characteristics of the freedom from interference for the action of interferences such as fluctuation noises is most important for tracking FM receivers. The questions of the effect of the interfering signals on the freedom from interference has not been extensively studied to this time.

Experimental results are known [1] which indicate that a tracking receiver has some selectivity and a high freedom from intereference for the action of interferences on a neighboring channel as compared with receivers in which a standard frequency demodulator (ChD) is used.

Some questions of the selectivity of a tracking demodulator with frequency feedback (OSCh) were studied in [2,3] for a non-ideal limiter and a nonlinear demodulation characteristic. The results led to the conclusion that for small interfering signal/useful signal ratios and for overlapping spectra of the signals the receiver in which a demodulator with OSCh is used cannot lower the cross interference.

In this study the selectivity characteristics were determined for a tracking receiver with SFD for the action of interfering signals.

<u>Harmonic interfering, low level signal</u>. The reaction of the useful and interfering harmonic signals with the SFD leads to the formation of a variable and constant component for the phase difference. For a small interfering signal/useful signal ratio ($U_п/U_c = k_a$ 0.3) the expression for the amplitude of the first harmonic ($Z_{пa}$) and for the constant component ($\bar{Z}_n$) for the phase difference are determined by the following expressions [4]:

$$Z_{na} = \frac{I_0^2(\eta)\,[M_0 M_1 \gamma_н \sqrt{1-\gamma_н^2} + \rho_1 \xi_{n0}]}{\sqrt{M_0^2(1-\gamma_н^2)+\xi_{n0}^2}}; \tag{1}$$

$$\bar{Z}_n = \frac{1}{2(1+\xi_{n0}^2)}\left[M_1^2 \gamma_н \sqrt{1-\gamma_н^2} - \frac{\rho_0 M_1 \xi_{n0}}{M_0}\right]. \tag{2}$$

Here $\gamma_н = \Omega_н/\Omega_{y0}$ is the relative initial frequency difference, $\xi_{n0} = \Omega_п/\Omega_{y0}$ is the relative frequency difference for the interfering signal, Ω_{y0} is the synchronization band of the SFD, $I_0(\eta)$ is a modified Bessel function of the first type of zero order, $\eta = M_1\sqrt{1-\gamma_н^2}/\xi_{n0}$; M_n, and ρ_n are the coefficients in the Fourier expansion of the functions

$$\sqrt{1+2k_n\cos\varphi + k_n^2} \approx M_0 + \sum_{n=1}^{N} M_n \cos n\varphi;$$

$$\frac{k_n(k_n+\cos\varphi)}{1+2k_n\cos\varphi+k_n^2} \approx \sum_{n=1}^{N} \rho_n \cos n\varphi,\ \varphi = \Omega_n t.$$

The amplitude for the output signal of the SFD is proportional to the phase difference $Z_{пa}$. Comparing (1) with the expression for the amplitude of the first harmonic for the oscillation at the output for the ChD with an ideal limiter shows that their difference is

determined by the effect of γ_{u}, in the SFD. For low values of γ_{u}, which occur in the majority of practical cases, the amplitude of the interference at the output of the SFD will be the same as at the output of a standard ChD. Consequently, for the action of small interferences which appear in the transmission band of a demodulator, the freedom from interference for a receiver with the SFD will be the same as in an ordinary receiver with ChD.

Selectivity of a tracking FM television receiver. The appearance of interference at the output of the SFD is explained by two reasons: first, at certain moments of time, for modulation, Ω_{n} falls in the transmission band of the receiver's output filter and secondly, the change in $\overline{Z}_{n}$ upon modulation.

From the practical standpoint the case in which the protective interval between the spectra for the wanted and interfering signals is quite big and interference does not occur because of the overlapping of the spectra is of greatest practical interest. For this, for high modulation indexes ($m > 3$) the following condition must be fulfilled

$$\xi_{n0} \geqslant \xi_{cp1}/2 + \xi_{cp2}/2 + \xi_{b}, \qquad (3)$$

in which $\xi_{cp1} = \Delta\omega_{cp1} / \Omega_{y0}$, $\xi_{cp2} = \Delta\omega_{cp2} / \Omega_{y0}$ are the ratios for the complete ranges for the change in the signal's frequency to the synchronization band.

In this case, the phase difference has an additional constant component (2) for the action of demodulated signals on the SFD.

For the frequency modulation of the interfering carrier this component for the phase difference changes and an interference arises at the output of the SFD whose level, generally, will depend not only on the characteristic of the interfering signal $k_n, \xi_n,$ but also on the modulation of the wanted signal.

For the transmission of an FM television signal in the interfering trunk, an interfering videosignal will be present at the output of the SFD according to (2) which is subjected to linear transformation.

If we consider that the discernment of the interfering videosignal and of the periodic low frequency interference have almost the same level [5], then in studying the interaction of the interfering and useful television signals in the receiver we can proceed from the standpoint of the heaviest conditions which correspond to a loading of the interfering trunk with a test television signal which is a sinusoidal, low frequency oscillation with an amplitude equal to half of the range of the videosignal from the black level to the white level. In addition, in order to provide for the permissible level of transmission of color television, it is necessary to consider the effect of the color subcarrier for the interfering signal. For the most serious case, it may be assumed that the level of the color subcarrier beyond the predistortion filter is equal to the full range of signal brightness.

For such a model for the interfering signal there will be a periodic interference at the output to the SFD whose level in the videochannel is determined by the ratio of the range for the videosignal between the black and

white levels to the maximum range of the interference

$$\psi_п = 20 \lg U_{бч}/U_{п\,макс}. \qquad (4)$$

The size of the protective ratio ψ_{ng} is approximated by the polynomial [5] for the monocrhomatic television

$$p_1(x) = 50 - 18x - 36x^2 + 17{,}64\,x^3 \text{ for } 0 < x < 1 \qquad (5)$$

and for color television

$$p_2(x) = 29{,}5 - 31{,}2\,x + 136{,}8\,x^2 - 108\,x^3$$
$$\text{for } 0{,}5 < x < 1, \qquad (6)$$

here $x = \omega/\omega_в$, $\omega_в = 2\pi \cdot 6 \cdot 10^6$ rad/sec.

The ratio of the signal to the periodic interference at the output of the tracking receiver is determined from the formula

$$\psi_п = 20 \lg (0{,}7\,\xi_{cp1}/Z_{пр} K_{вф}), \qquad (7)$$

where $Z_{пр}$ is the doubled amplitude for the change in the phase difference due to modulation of the interfering signal, $K_{вф}$ is the transmission coefficient for the restoring filter at the interference frequency.

We must keep in mind that the periodic interference causes the greatest interfering action at the levels for (25-40%) of the videosignal's range. The total range for the change in the phase difference in SFD does not exceed 1 rad for a complete range for the change in

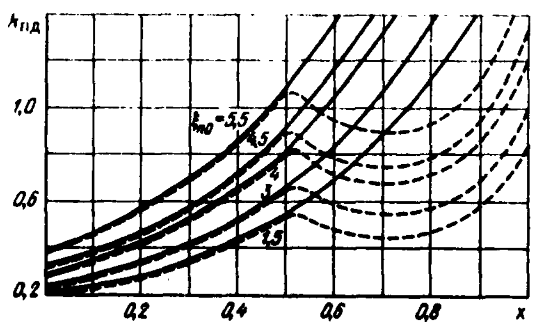

Fig. 1

the signal's frequency but the gray level is transmitted by a signal frequency close to the carrier frequency. Therefore, we can assume, approximately, that $\gamma_{\text{ж}} = 0$ and use the following formula for the calculation:

$$\overline{Z}_{\text{п}} \approx k_{\text{п}}^{2} \xi_{\text{п}}/2(1 + \xi_{\text{п}}^{2}). \qquad (8)$$

For the modulation of the interfering signal with a sinusoidal signal, the relative frequency $\xi_{\text{п}}$ will change from the minimum value

$$\xi_{\text{п мин}} = \xi_{\text{п}0} - \xi_{\text{ср}2}[0{,}0804 - 0{,}7\, K_{\text{пф}}(x)]/2 \qquad (9)$$

to the maximum value

$$\xi_{\text{п макс}} = \xi_{\text{п}0} + \xi_{\text{ср}2}[0{,}0804 + 0{,}7\, K_{\text{пф}}(x)]/2, \qquad (10)$$

simultaneously with the synchropulses. Here $K_{\text{пф}}$ is the transmission coefficient for the predistortion filter.

The value of the protective ratio $k_{\text{пд}}$ is found from (5) - (10):

$$k_{\text{пд}} = \frac{1}{10^{\psi_{\text{пд}}(x)/20}} \times$$
$$\times \sqrt{\frac{1{,}4\,\xi_{\text{ср}1}}{K_{\text{вф}}(x)\,[\xi_{\text{п макс}}/(1 + \xi^{2}_{\text{п макс}} - \xi_{\text{п мин}}/(1 + \xi_{\text{п мин}})^{2}]}}. \qquad (11)$$

The relationships $k_{\text{пд}}(x)$ (solid lines - for monochromatic television, dotted curves for color television) are given in Fig. 1. These curves show that for $0 < x < 0.5$, $k_{\text{пд}}$ increases with an increase in x. For $0.5 < x < 0.8$ (region for the color sub-carrier) $k_{\text{пд}}$ for the color television

is less than for black and white. The lower the frequency for the test signal the lower the value of $k_{\text{пд}}$ which corresponds to it for low values of x and the values of $k_{\text{пд}}$ close to the color subcarrier differ by 6 dB. In calculating, the case for the complete exclusion of the effect of an interfering signal should be used as a guide, therefore the values of $k_{\text{пд}}$ for low values of x should be used. The dependence $k_{\text{пд}}$ ($\xi_{\text{по}}$) for x = 0.05 is shown in Fig. 2. It characterizes the selective properties of the SFD for the action of an interfering television FM signal. This dependence can be used as the required protective interfering signal/useful signal ratio in a tracking receiver. If k_n exceeds $k_{\text{пд}}$ then an intermediate frequency channel must provide the additional selectivity for the receiver.

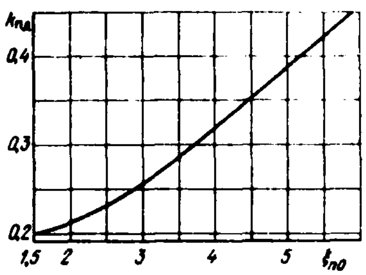

Fig. 2.

Selectivity of a tracking FM receiver for multichannel telephony. Let us consider the case in which the distribution of the carrier frequencies between the tracks is so large that if the frequency of the interfering signal is changed the interference cannot fall into the receiver's transmission band. For large modulation indexes (m > 3) it is obviously necessary that the following condition by satisfied

$$\xi_{n0} \geqslant \Delta\xi_c x_c + \Delta\xi_n x_n + \xi_{\text{в}}, \qquad (12)$$

In which x_c and x_n are, respectively, the peak factors for a multichannel useful and interfering signal, $\xi_{\text{в}} = \Omega_{\text{в}} / \Omega_{y0}$ — is the normalized upper limit frequency for a multichannel signal.

The power of the signal for multichannel communications at a point with a measuring level p_k is equal to

$$P_c = e^{2p_k + 2p_{cp}} = \frac{\Delta\omega^2_{дсэ}}{\Omega_{y0} R_k} S_{\phi д} K^2_{ry}, \qquad (13)$$

here $\Delta\omega_{дсэ}$ is the effective deviation in the frequency of the useful signal, K_{ry} is the transmission coefficient for the group amplifier, P_{cp} is the level of the mean power of the multichannel communication, and R_k is the load resistance at the point of measurement.

The power of the interference in the kth channel taking (13) into account is

$$P_п = \frac{k^2_{пс} W_k G_2(\sigma_k) K^2_{ry}}{R_k} = \frac{e^{2p_k} K^2_{пс} W_k \delta^2 G_2(\sigma_k)}{\Delta\xi_{дпк}}, \qquad (14)$$

here K_{nc} is the psophometric coefficient, W_k is the band frequency of the telephone channel, $G_2(\sigma_k)$ is the spectral density of the power of the phase difference $\delta = \Delta\xi_{дпэ} / \Delta\xi_{дсэ}$ is the ratio of the effective deviations in the frequency for the interfering and useful signals, $\Delta\xi_{дпк}$ is the effective deviation for the frequency of the interfering signal in the channel.

To find $G_2(\sigma_k)$ we expand the expression for the phase difference $\bar{Z}_n(\xi_n)$ in a power series for the relativity of the point ξ_{n0} and we will limit ourselves to the first three terms of the expansion.

Then we will determine the spectral density of the interference by means of the Fourier transformation from the correction function. Dropping the calculation by

means of the method in [7], we will write the expression for the power of the interference in the telephone channel with a relative level of zero

$$P_{ш} = 10^{9} \frac{k_{пс}^{2} W_{k} k_{п}^{2} \delta^{2} e^{2p_{ср}}}{2W} \left\{ \left[\frac{A}{B^{4}} + 36 \frac{C}{B^{8}} e^{4p_{ср}} \Delta\xi_{дпк}^{4} + \right.\right.$$

$$\left. + 12 \frac{AC}{B^{6}} e^{p_{ср}} \Delta\xi_{дпк}^{2} \right] y_{1}(\sigma_{k}) + 8 \frac{D^{2}}{B^{6}} e^{2p_{ср}} \Delta\xi_{дпк}^{2} y_{2}(\sigma_{k}) +$$

$$\left. + 24 \frac{C^{2}}{B^{8}} e^{4p_{ср}} \Delta\xi_{дпк}^{2} y_{3}(\sigma_{k}) \right\}. \qquad (15)$$

где $A = 1 - \xi_{п0}$; $B = 1 + \xi_{п0}^{2}$; $C = \xi_{п0}^{5} - 4\xi_{п0}^{4} -$
$- 3\xi_{п0}^{3} + 20\xi_{п0}^{2} - 3\xi_{п0}$; $D = \xi_{п0}(3 - \xi_{п0}^{2})$;

for calculating the system of transmission with multichannel telephony. Here $\sigma_{k} = F_{k} / F_{в} - F_{н}$; F_{k} is the channel frequency, $y_{1}(\sigma_{k})$ is the coefficient equal to one in the frequency band $(F_{B} - F_{H})$ and zero outside this band, $y_{2}(\sigma_{k})$ and $y_{3}(\sigma_{k})$ are coefficients which take into account the power distribution for the products of nonlinear distortion [7].

Being given a certain value for the power of the interference $P_{п}$, equation (15) can be used to determine the permissible protection ratio $k_{пд}$. The value of $P_{п}$ must be found from the general standards for the power of transmission noises in the system. A calculation was made for $k_{пд}$ using equation (15) for $P_{п}$ = 500 pW for transmission by 600 channel telephony in a two track system. In the calculation it was assumed that the effective frequency deviation for the FM signals are the same and the effective

deviation for the phase difference with the SFD is equal to 0.2 rad. The dependence $k_{пд}(\xi_{п0})$ is shown in Fig. 3. From this we can see that the SFD has a high selectivity.

We note that in a standard ChD with an ideal limiter and for non-overlapping spectra for the wanted and interfering signals there is no interference but its occurence in the SFD is basically unavoidable. Therefore, in order to attenuate the interference in the neighboring channel, a filter or a limiter must be introduced in front of the SFD. If a limiter is used the amplitude modulation suppression must be close to ideal [8] or else this may lead to an increase in the interference's power to a value greater than provided for by the SFD.

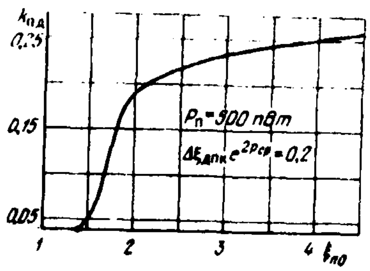

Fig. 3

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Drop in the Threshold Noise Levels of FM Receivers With Postdetector Treatment of the Signal

Yu. I. Tarakanov

The basic methods for building FM demodulators are studied in which postdetector treatment of the signal is used to lower the effect of the threshold component of the noise. The results achieved in studying and using such demodulators are described.

The study of the postdetector treatment of the signal to lower the noise level at the output of FM receivers during operation in the range of signal/noise ratios lower than the threshold may be desirable in many cases in view of the great stability of the low frequency circuits, their lack of dependence on the value of the intermediate frequency, and the fact that numerical methods of treatment can be used. Let us consider the basic results which were obtained in studying and using postdetector treatment of the signals to lower the level of the threshold noises at the output of FM receivers.

All of the methods for lowering the threshold by post detector treatment of the signal can be divided into three groups. The first group is made up of demodulators based on observing the threshold pulses at the output of the ordinary frequency detector with a broad band output circuit and their subsequent elimination from the output voltage of the demodulator. This principle for the action of a demodulator with postdetector treatment of the signal has been described in [1-5]. The second group of demodulators with post detector treatment of the signal contains the video analogs of high frequency, threshold lowering demodulators.

Let us list the known ways of using the postdetector treatment and we will show a method which is based on observing the threshold pulses at the output of the ordinary frequency detector with a broad band output circuit followed by their elimination from the output voltage of the demodulator [1-5]. The method of creating video analogs of high frequency threshold lowering devices is another way. The operation of these low frequency devices is described by the same equations as is the operation of the corresponding high frequency demodulators. The analogy between the demodulators which operate at low and at intermediate (or high) frequency was observed in [6, 7]. The functional circuits of the video analogs of the synchrono-phase demodulator and for a demodulator with frequency feedback were given in [8]. Nonlinear elements witha periodic characteristic make up these functional circuits. The voltage from two detectors, a frequency (or phase) and amplitude detector, are used in all of the devices with postdetector treatment, which were described in [6, 10].

Considering that for input signal/noise ratios lower than the threshold the useful information is included both in the instantaneous phase and in the amplitude sum for the FM signal and the noise [11] we can point to the approach to post-detector treatment both for the problem of the optimum use of the information and for minimizing the noise power at the output of the FM receiver.

Let us dwell on some of the demodulators with postdetector treatment of the signal which were studied.

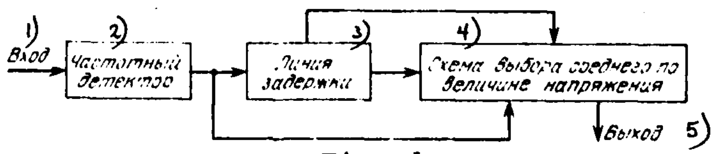

Fig. 1

1) input, 2) frequency detector, 3) delay line, 4) circuit for selecting the mean voltage, 5) output

The functional scheme for a demodulator which eliminates the threshold pulses from the output voltage of the frequency detector is shown in Fig. 1. In contrast to the demodulators described in [1-5], there is neither a special device for observing the threshold pulses nor a predicting filter in the scheme. The frequency detector with a low time constant for the output circuit is connected with the delay line, whose output voltages are a period of τ and 2τ behind the input voltage. The value of τ exceeds the duration of the pulses being suppressed but is much less than the period of frequency modulation, i.e., $(1/B) < \tau \ll (1/F_B)$ where B is the transmission band for the track which proceeds the demodulator and F_B is the top frequency of the spectrum for the incoming communication. When this condition is fulfilled the scheme for selecting the mean voltage, at the outlet of which at every moment of time the instantaneous value of one of the three input voltages is observed, suppresses the threshold pulses except for those which follow one another with an interval no greater than 2τ and which have the same polarity.

The value of τ is limited by the level of the nonlinear distortions which arise in the scheme for selecting the mean voltage. For a sinusoidal voltage at the output to the frequency detector these distortions can be readily evaluated without taking into account the action of interferences: $u = U \sin \Omega t$. Let us expand the periodic voltage at the demodulator's output in a Fourier series. Then for $\Omega\tau \ll 2\pi$, the expansion coefficients are equal to:

$$a_1 \approx U; \quad a_n \approx \frac{2U(\Omega\tau)^2}{n\tau} \sin \frac{n\Omega\tau}{4} \sin^2 \frac{n\pi}{2}, \quad n=2, 3, 4, \dots$$

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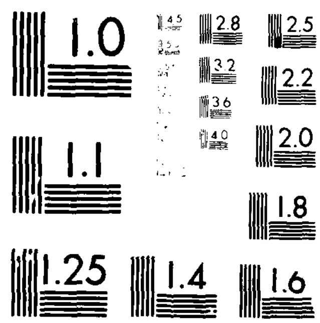

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and the coefficient for the nonlinear distortions is

$$k_r = \frac{1}{a_1}\left(\sum_{n=2}^{l} a_n^2\right)^{1/2}.$$

Summation is made for all of the harmonics which fall into the frequency band for the communication received [0, F_B] since the rest of the harmonics will be filtered out by the band filter at the output to the demodulator. For wide band FM the time τ can be selected so that the condition for the suppression of the threshold pulses $1/B < \tau$ will be observed for a low level of nonlinear distortions.

Let us consider the optimization of the postdetector treatment of the signal in an FM receiver. We will assume that the incoming communication has a constant spectral density in the frequency range [0, F_B] and an even probability distribution rule. We know that for the addition of Gaussian noise with a symmetrical spectrum and with a dispersion σ^2, to a sinusoidal signal with a circular frequency ω_C, the conditional distribution density for the instantaneous frequency ω_Σ for the total voltage with an envelope U_2 is equal to [12]

$$p(\omega_\Sigma / U_\Sigma) = \frac{1}{\sqrt{2\pi}\sigma_\omega} e^{-\frac{(\omega_\Sigma - \omega_c)^2}{2\sigma_\omega^2}}.$$

where $\sigma_\omega^2 = \sigma^2 \gamma^2 / U_\Sigma^2$ is the dispersion for the noise addition to the signal's frequency, γ is the rotation radius of the spectral density of the input noise. If we assume that the conditional distribution law $p(\omega_\Sigma / U_\Sigma)$ is retained for a chane in the signal's frequency during modulation by $\Delta\omega$ and that $\Delta\omega$ is invariable during the time interval $1/2F_B$, then we can evaluate the maximum probability (which in this case coincides with the evaluation in terms of the maximum

a posteriori distribution density) by means of the ordinary methods of statistical theory

$$\Delta\omega = \frac{\sum_{i=1}^{n} \omega_{\Sigma i}/\sigma^2_{\omega i}}{\sum_{i=1}^{n} 1/\sigma^2_{\omega i}},$$

where $\omega_{\Sigma i}$ and $\sigma^2_{\omega i}$ are random values of ω_Σ and the corresponding values for the dispersion σ^2_ω . Considering that σ^2_ω is inversely proportional to U^2_Σ, we can express the functional scheme for the demodulator (Fig. 2) which gives the optimum evaluation of $\Delta\omega$ according to the last expression.

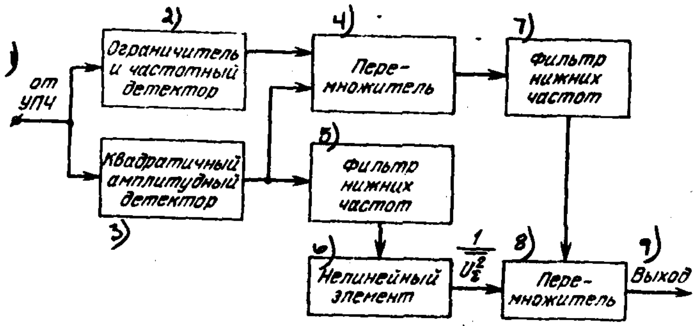

Fig. 2

1) From the UPCh, 2) Limiter and frequency detector, 3) Quadratic amplitude detector, 4) Cross multiplier, 5) Low frequency filter, 6) Nonlinear element, 7) Low frequency filter, 8) Cross multiplier, 9) Output

In this scheme the low frequency filter which follows the remultiplier carries out a weighted summation of the values of ω_{i}, the greater weight being given for the lower σ^2_ω. This operation coincides with that which takes place in a demodulator described in [6, 9]. The additional elements, as compared with that demodulator (the second low frequency filter, the nonlinear element with a hyperbolic characteristic which produces a voltage inverse to the averaged value of the

square of the envelope U_Σ^2 and the output remultiplier) decrease the action of the noises on the output amplitude detector which take place for a deviation of $\Delta\omega$.

The possibility of lowering the noise level at the output to the receiver by means of demodulators with postdetector treatment of the signal was confirmed experimentally.

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Switch Model for Noise in an FM Receiver and Lowering of the Threshold for FM Reception

L.P. Yaroslavskiy

A statistical model is formulated for noise in a standard FM demodulator which generalizes, in analytical form, the known theoretical and experimental facts and it is discussed in application to the synthesis of threshold-lowering schemes for FM reception.

Let u(t) be the process at the output of the frequency discriminator, $\lambda(t)$ be the signal modulation (communication) as a function of time $(\lambda \in [-1/2, 1/2])$.

The following model for the noise in an FM receiver follows from the results of the experimental study of the statistical characteristics of the interference in FM reception [1, 2] and the analytical solution of the problem of the distribution and the spectrum for the derivative phase for the sum of a sinusoidal signal and a narrow band Gaussian noise [3, 4].

$$u(t) = \lambda(t) + [1 - a(t)]\, n_H(t) + a(t)[kn_c(t) + p(t)]. \tag{1}$$

Here $n_H(t)$, $n_C(t)$ are the derivative sine and cosine components of the input noise, p(t) is some pulsed chance process, a(t) is a switching chance process which assumes the values 0 and 1 and which is not correlated with $n_H(t)$, $n_C(t)$, and p(t). The most important parameters which characterize the suggested model are the average value of the switching process $\bar{a}$ the power of the correlation functions, and the mean values for the processes $n_H(t)$, $n_C(t)$, and p(t), and also their mutual correlation functions. By making use

of the results for the mean value of the derivative phase the sums of the sinusoidal signal and Gaussian noise [3] and having written the expression for the correlational function of the derivative phase, we can obtain the following expressions (using S. Rice's designations [4]):

$$\overline{a} = e^{-\rho}; \ \overline{p(t)} = -\lambda; \ k = \sqrt{2\rho}\,\lambda; \tag{2}$$

$$\overline{n_{н}(t)} = \overline{n_c(t)} = 0; \quad \overline{n_c(t)\,p(t+\tau)} = \overline{n_{н}(t)\,p(t+\tau)} = 0; \tag{3}$$

$$\overline{n_{н}(t)\,n_{н}(t+\tau)} = -\frac{1}{2\rho}\,g''(\tau); \ \overline{n_c(t)\,n_c(t+\tau)} = g(\tau), \tag{4}$$

and also the expantion $\widetilde{p(t)p(t+\tau)}$ into a binary series, which contains members with the form $g''(\tau)gk(\tau)h(\tau)$;

$$h''(\tau)\,g^k(\tau)\,h^l(\tau); \quad g^k(\tau)\,h^l(\tau); \quad g'(\tau)\,g^k(\tau)\,h^l(\tau);$$
$$h'(\tau)\,g^k(\tau)\,h^l(\tau); \quad g'^2(\tau)\,g^k(\tau)\,h^l(\tau); \quad g'(\tau)h'(\tau)g^k(\tau)h^l(\tau);$$
$$h'^2(\tau)\,g^k(\tau)\,h^l(\tau), \quad k,\ l = 0,\ 1, \ldots,$$

where $g(\tau)$ and $h(\tau)$ are determined by the relationship

$$g(\tau) + ih(\tau) = \frac{\int\limits_0^\infty w(f)\exp\left(2\pi i\,\frac{f-f_0-\lambda\,\Delta f}{\Delta f}\,\tau\right)df}{\int\limits_0^\infty w(f)\,df};$$

$w(f)$ is the noise energy spectrum at the input of the discriminator;
f_0 is the frequency of symmetry $w(f)$;
Δf is the full deviation of signal frequency.

The pulse process $p(t)$, in turn, can be presented in the form of two components (anomalous and subanomalous), which are switched by a certain process $s(t)$:

$$p(t) = s(t)\,p_a(t) + [1 - s(t)]\,p_{ca}(t), \tag{5}$$

whereby $\overline{p_a(t)} = -\lambda$; $\overline{p_{ca}(t)} = 0$ and they all are not correlated to one another and to the remaining components of the model. The empirical data of the energy spectra $p_a(t)$ and $p_{ca}(t)$ are given in [1, 2]. The empirical relation of s(t) to λ and p, is constructed from data (data presented by M.A. Rabinovich), produced on a digital model of an FM demodulator, which is shown in Fig. 1.

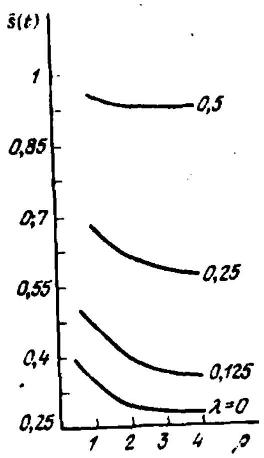


Fig. 1

A model, similar to (1), can be used also for describing a signal at the output of the low-frequency filter of a standard demodulator of FM signals. In this case, relationships (2) and (3) are preserved, but (4) is replaced by its convolution with the pulse reaction of the low-frequency filter. In practice it happens that the subanomalous component at the FNCH output is absent, and the spectrum of the anomalous component equals the square of the modulus of the frequency characteristic of the FNCH.

Such a model can be used for describing the noise at the output of a frequency discriminator and in a receiver with frequency feedback. In this case one must take into account that λ is the difference of the transmitted frequency and its evaluation at the output of the feedback loop, which contains a certain random component.

By using model (1), we can suggest a method for lowering the threshold during reception of FM signals, which is based on the principle of detection and suppression of anomalous pulses of noise [5, 6].

Insofar as available data permit one to find only the energy spectra of noise components in an FM demodulator, an anomalous pulse detector must be constructed as an optimal detector of the pulse signal (in the given case of anomalous pulses) on a background of colored noise (in the given case, communications of Gaussian and subanomalous noise). Such a detector consists of an optimal linear filter and a threshold device, which records a pulse when a signal at the output of an optimal filter exceeds a threshold level, which is selected, for example, from the condition of equal probability of passage and false detection (or of any other similar condition). In order to find the frequency characteristics for the optimum filter and to evaluate the amplitude spectrum for the anomalous pulses we can use a function which is equal to the square root of their energy spectrum.

The measuring device $\lambda(t)$ for $a(t) = 1$ must be built taking into account the fact that at the moment anomalous pulses occur the communication is totally suppressed. Therefore, the best measuring device is one that makes an optimum prediction of the values of $\lambda(t)$ for $a(t) = 1$ from its value of neighboring moments of time when $a(t) = 0$. Such a prediction is best made by using communication which has already been filtered. Sometimes it is simpler in practice to compensate for the anomalies by introducing compensating pulses into the signal being filtered for $a(t) = 1$ rather than to suppress them.

Schemes which realize the given principle for the observation and suppression of anomalies are shown in Figs. 2, 3, and 5. These differ in the point of their connection in the demodulator and by the different use of the a priori information on communications and interferences [5].

<u>Filtration of anomalies at the output of the FNCh.</u> The scheme for a filter for anomalies, which is connected at the output of the NCh filter is shown in Fig. 2 [5]. This scheme can be used to filter out anomalies both in a standard frequency detector and in a detector with frequency

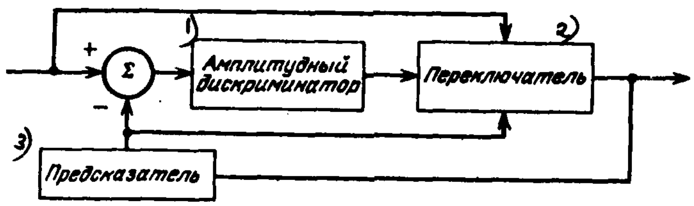


Fig. 2

1)Amplitude discriminator, 2) Switch, 3) Predictor

feedback if the communication is highly correlated within the limits of the duration of the FNCh's response. Its modeling on a computer in the case of image transmission showed that it shifts the threshold curves with respect to the input signal/noise ratio by (4-5 dB).

<u>Filtration of anomalies at the output to the frequency discriminator</u>. Version 1 (Fig. 3). The scheme which is shown in Fig. 3 can be used in a standard frequency detector in the case in which the communication spectrum $\lambda(t)$ is not known and only its frequency band width is known. The threshold curve, which is obtained for modeling this scheme on a computer (with some variations) for a coefficient

of expansion of the band $\beta = 8$ is shown in Fig. 4 [7].

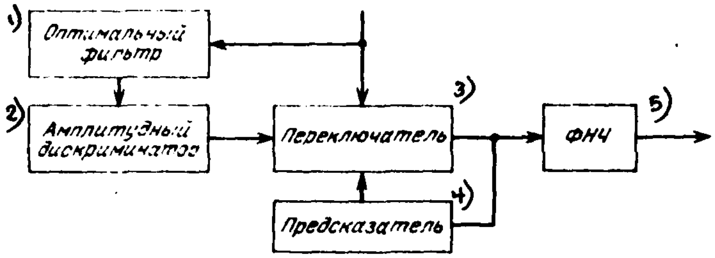
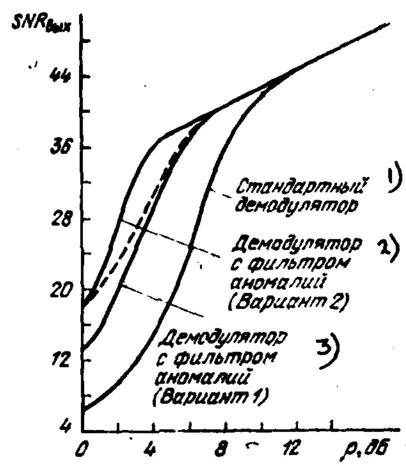


Fig. 3

1) Optimum filter, 2) Amplitude discriminator, 3) Switch, 4) Predictor, 5) FNCh



1) standard demodulator
2) demodulator with anomaly filter (Version 2),
3) demodulator with anomaly filter (Version 1)

Fig. 4

Version 2 (Fig. 5). This scheme can be used if the energy spectrum of the communication is known in which case the range of correlation for the communication is greater than the response time of the FNCh. In this case, it is best to use the low frequency part of the anomaly spectrum and the effectiveness of observing them is increased. The threshold curve which is obtained for modeling such a system on a computer for $\beta = 8$ is shown in Fig. 4 [7].

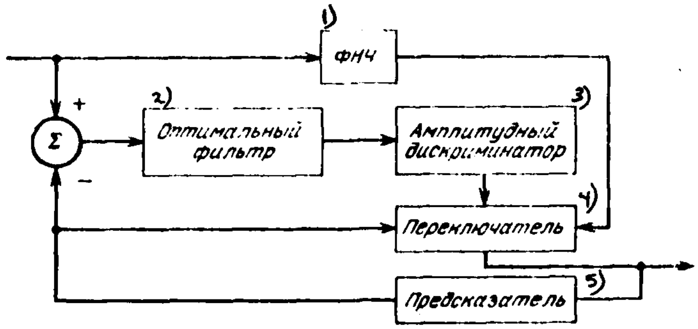


Fig. 5

1) FNCh, 2) Optimum filter, 3) Amplitude discriminator, 4) Switch, 5) Predictor

Let us compare the freedom from interference which is provided by the methods of observing and suppressing anomalies with the potential freedom from interference for FM reception. We can evaluate the potential freedom from interference for FM reception for communications with an even spectrum within the limits of the communication band by using the analogy between VIM and FM [8]. The result of evaluating the output signal/noise ratio for an ideal FM receiver, using the formula

$$\mathrm{SNR}_{вых} = \frac{1 - P_a}{\sigma_н^2 (1 - P_a) + \frac{1}{12} P_a}\,.$$

is shown in Fig. 4 (dotted line). Here $P_a = (\beta/2\sqrt{6}) e^{-\rho\beta/4}$ is the probability of anomalous errors in metering the communication, $\sigma_н^2 = 1/3\beta^2\rho$ is the signal/noise ratio for "normal" noise.

Thus, it appears that the scheme (Fig. 3) for filtering the anomalies at the output to the discriminator is close to "ideal" and the scheme (Fig. 5) is better. This is explained

by the fact that in this case the correlation of the communication within the limits of the Kotel'nikov-Naykvist range is used and the evaluation for "ideal" reception is found without taking this correlation into account.

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Analysis and Comparison of Multifilter Methods of Reception

A.I. Zodzishskiy,
A.A. Kiy,
V.A. Sakharov,
V.P. Sokolov

The comparative characteristics were studied for multifilter systems with the filters connected in parallel and with the automatic selection of filters in application to the problem of narrow band filtering of a harmonic signal whose frequency is not accurately known. A method of calculating the required number of filters is developed and of determining the area for the preferable use of both schemes depending on the given signal/noise ratios and the probability of the proper switching on of the filters.

It is necessary to filter a harmonic signal from the noise in many problems of radio technology in which the signal's frequency is not precisely known and is variable. Usually either a narrow band tracking filter or a device which consists of a group of nontracking, narrow band filters with overlapping frequency ranges are used for this purpose. The multifiltering schemes have certain advantages over the tracking filters, the main one is the fast connection which is particularly important when working with intermittent signals.

Today two types of multifiltering schemes have been widely used: a scheme with parallel connected filters and a scheme with the automatic selection of the filter in whose transmission band the signal is to be found [1-3, 6]. One of the schemes with parallel connected filters is shown in Fig. 1. The group of band filters ϕ_i (i = 1, 2,...,N) overlaps the range of

unknown frequencies forming N channels. In each of the channels there is an envelope detector D after the filter whose output voltage is comparable with the threshold a. If the envelope for the process exceeds the threshold a, then the oscillations beyond the channel filter pass through the commutating cascade (KK) and go into the output of the circuit.

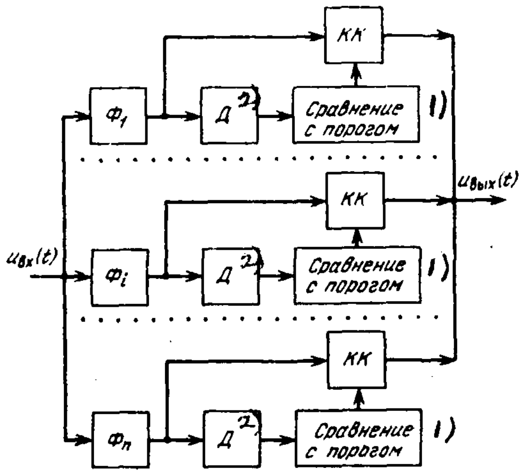

Fig. 1

1) Comparison with threshold; 2) D

In the scheme with automatic selection (Fig. 2) the input signal also passes through a group of narrow band channel filters Φ_i.

There is a detector at the output for each channel which separates out the envelope of the process. The scheme for selecting the maximum (SVM) by comparing the values of the envelopes, automatically opens the commutating cascade for

that channel in which the envelope is maximal, blocking the other channels in so doing.

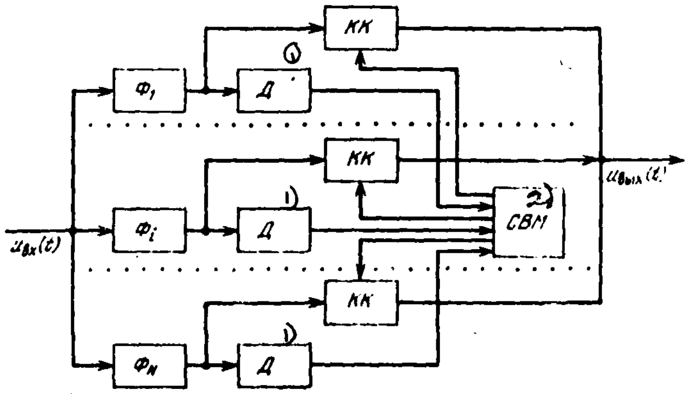

Fig. 2

1) D ; 2) SVM

The specific feature of the multifiltering schemes is that the channel, in whose band the signal is found, can be switched off of the circuit's output from time to time due to the action of the noises. The probability for such an event $P_{\Pi p}$ will be called the probability for signal passage.

The operation of the multifiltering schemes can be characterized quite completely by means of two output parameters: the ratio of the signal's power to the noise's power at the output from the circuit $(P_c/P_ш)_{вых}$ and the probability for the signal's passage $P_{\Pi p}$. In this case $(P_c/P_ш)_{вых}$ will determine the so-called "smooth" fluctuations in the output signal and $P_{пр}$ will determine the probability of the appearance of large scatterings (anomalous errors). Let us find these parameters for the given schemes.

Let the input voltage $U_{вх}(t)$ (Figs. 1 and 2) be an additive mixture of a harmonic signal $S(t) = U_0\cos(\omega_c t + \varphi_c)$ and a normal stationary noise n(t) with an even spectral density G_0 in the band Δf_Σ, which is overlapped by N channel filters. We will assume that the amplitude-frequency characteristics of the channel filters have a rectangular shape which do not overlap and they have the same transmission bands equal to $\Delta f = \Delta f_\Sigma/N$ and that the power of the noise in the band of each filter is $\sigma^2 = G_0\Delta f$. If we assume that the processes x(t) at the output of the channel filters are narrow band processes they can be represented in the form

$$x_i(t) = A_i(t)\cos[\omega_i t + \varphi_i(t)],$$

where $A_i(t)$ and $\varphi_i(t)$ are the envelope and phase of the process, ω_i is the tuning frequency of the i-th channel filter.

For a scheme with parallel connection $P_{пр}$ is the probability that the envelope signal and noise at the output of the filter in whose transmission band the signal is found, drops below the threshold a

$$P_{пр\,п} = \int_0^a W_c(A)\,dA, \tag{1}$$

where $W_c(A) = \frac{A}{\sigma^2}\exp\left\{-\frac{A^2+U_c^2}{2\sigma^2}\right\}I_0\left(\frac{AU_c}{\sigma^2}\right)$

is the Rayleigh-Rice distribution law.

For a circuit with the automatic selection $P_{прА}$ is equal to the probability that the envelope for the signal and noise mixture at the output to the filter with the signal will be less than the maximum value for the envelope A_{max} at the

output of the other filters. Since A_{max} is, in essence, a floating threshold, an additional averaging by means of the distribution rule $W_c(A_{max})$ is needed for calculating $P_{\pi pA}$:

$$P_{np\,A} = \left\langle \int_0^{A_{max}} W_c(A)\,dA \right\rangle \qquad (2)$$

It is obvious that $P_{np\,A}$ is the probability that A_{max} exceeds the value of the envelope in the filter with the signal. Taking into account the fact that the processes in the channel filters are independent and also that the envelope in the channels without the signal is distributed according to Rayleigh's law and it is distributed according to the Rayleigh-Rice law in the channel with the signal and by using [4, p. 67] we have

$$P_{np\,A} = 1 - \frac{1}{N}\sum_{i=1}^{N}(-1)^{i-1} C_N^i\, e^{-\gamma^2(i-1)/i}. \qquad (3)$$

Here $\gamma^2 = (P_c/P_{ш})_{вх} N$ is the signal/noise ratio in the band of one filter.

The results of the calculations made on the M-222 computer and using equation (3) are given in the appendix.

In some cases approximate expressions can be used in place of (3). Since $P_{\pi pA}$ is described by the sum of the sign-variable series, then by using only the first term of the series in (3)

$$P_{np\,A} \simeq \frac{N-1}{2}\, e^{-\gamma^2/2}, \qquad (4)$$

we overcome the error of the approximation

$$\delta P_{np\,A} = \frac{\Delta P_{np\,A}}{P_{np\,A}} \leqslant \frac{N-2}{3}\, e^{-\gamma^2/2}.$$

For example, $\delta P_{np\,A} \leqslant 0{,}06$ for $N = 14$ for $\gamma^2 = 24$ and $\delta P_{np\,A} \leqslant 0{,}0045$ for $\gamma^2 = 48$.

We should point out that for the circuit with automatic selection, if the output from a filter whose transmission band contains the signal, is connected to it, all of the remaining channel filters (N-1) must be closed but for the circuit with parallel connection several filters can still be open.

The $(P_c/P_w)_{вых}$ ratio only has meaning for the circuits shown in Figs. 1 and 2 if the output from a filter containing the signal is connected to them. For circuits with parallel connection

$$(P_c/P_w)_{вых\,п} = \frac{P_c}{P_{w\Sigma} + P_{wc}},$$

in which $P_{w\Sigma}$ is the mean power of the noises which pass through the output of the circuit along the channels which do not contain the signal and P_{wc} is the mean power of the noises which pass through the channel with the signal.

We will assume that $P_{wc} = \sigma^2$. Strictly speaking, for $a < U_c$ which is almost always the case, P_{cw} is somewhat less than σ^2. If fact, the channel with the signal is blocked at that moment for which the envelope becomes less than the threshold because of the action of the noise and at that moment the instantaneous power of the noise is greater than its mean value.

The well known relationship of the noise's power on the mean square of its envelope [5], $P_w = \langle A^2\rangle/2$, is used to determine the mean power of the noise that passes to the circuit's output from one of the channels which does not contain the signal.

Since the noise only passes into the circuit's output for the condition that its envelope exceeds the threshold a then

$$P_{\text{ш}i} = \frac{< A^2/A > a >}{2} T_{\text{отн}} = \frac{1}{2} T_{\text{отн}} \int\limits_a^\infty A^2 W(A/A \geqslant a) dA. \tag{5}$$

Here $T_{\text{отн}} = \int\limits_a^\infty W(A)\, dA$ is the relative time that the envelope stays over the threshold a;

$$W(A/A \geqslant a) = \begin{cases} \dfrac{W(A)}{\int\limits_a^\infty W(A)\, dA} & \text{for } A \geqslant a, \\ 0 & \text{for } A < a \end{cases}$$

is the conditional rule for the envelope's distribution,

$$W(A) = \frac{A}{\sigma^2} \exp\left(-\frac{A^2}{\sigma^2}\right)$$

is the unconditional rule for the envelope's distribution.

By making the necessary transformations, we find

$$P_{\text{ш}i} = \sigma^2 \left(1 + \frac{a^2}{2\sigma^2}\right) \exp\left(-\frac{a^2}{2\sigma^2}\right). \tag{6}$$

Taking into account that the total number of channels without a signal is equal to N-1 and that the noises in them are independent we have

$$P_{\text{ш}\Sigma} = (N-1) P_{\text{ш}i}.$$

Thus, for the circuit with filters connected in parallel we get

$$(P_c/P_{\text{ш}})_{\text{вых п}} = (P_c/P_{\text{ш}})_{\text{вх}} N \times$$
$$\times \left[1 + (N-1)\left(1 + \frac{a^2}{2\sigma^2}\right) \exp\left(-\frac{a^2}{2\sigma^2}\right)\right]^{-1}. \tag{7}$$

In the circuit with automatic selection when the filter with the signal is switched on $(P_C/P_Ш)_{вых}$ coincides with the signal/noise ratio in the band for this filter and it is equal to

$$(P_c/P_ш)_{вых\,A} = \gamma^2 = (P_c/P_ш)_{вх}\,N. \qquad (8)$$

Here, as in the preceding case, the slight decrease in the noise's power due to the fact that for a low enough value for the probability of $P_{пр}$, the blocking of the channel with the signal takes place predominantly at a low level for the envelope in this channel is not taken into account.

In practice, the multifilter circuit is designed on the basis of a given value for the input ratio $(P_C/P_ш)_{вх}$ and the permissible values of the output parameters $(P_C/P_ш)_{вых}$, $P^*_{пр}$.

The design of the circuit with parallel connection is reduced to finding the necessary number of filters N and the threshold a. The circuit with automatic selection has only one degree of freedom - the number of filters N and therefore its design is reduced to determining just this number. The values which are found should be such that they provide for the fulfillment of the following conditions:

$$P_{пр} < P^*_{пр}, \qquad (9)$$

$$(P_c/P_ш)_{вых} \geqslant (P_c/P_ш)^*_{вых}. \qquad (10)$$

The complexity of creating multifilter circuits depends mainly on the number of filters which are used in it. Therefore, it is desirable that the number of filters be as low as possible.

For circuits with parallel connection the value of N and a can be determined if the system of inequalities (9) and (10) are reduced to equalities and if we substitute equations (1) and (7) in them. For circuits with automatic selection, after substituting equations (3) and (8), which, as before, are considered as equalities, in (9) and (10), we get two equations with a single unknown. Two values of N can be found from solving these equations. It is obvious that the larger of the two is the one we want.

The method which has been described for determining the required number of filters analytically is associated with great mathematical difficulties. Therefore we will use the following method. We will construct a group of curves which express the relationship $P_{\text{пр}} = f\left\{(P_c/P_{\text{ш}})_{\text{вых}}\right\}$ for different numbers of filters N. Such a group for the circuit with a parallel connection and using the tables for the integral Rayleigh-Rice distribution law for $(P_c/P_{\text{ш}})_{\text{вх}} = 1$, is shown in Fig. 3. The form of the curves is determined by the value of the threshold a established in the circuit. As a is increased the ratio $(P_c/P_{\text{ш}})_{\text{вых}}$ increases with the simultaneous increase in the probability for the passage of the signal $P_{\text{пр}}$. As a is increased $P_{\text{пр}} \longrightarrow 1$ and $(P_c/P_{\text{ш}})_{\text{вых}} \longrightarrow (P_c/P_{\text{ш}})_{\text{вх}}$ N. For a $\longrightarrow 0$, $P_{\text{пр}} \longrightarrow 0$ and $(P_c/P_{\text{ш}})_{\text{вых}} \longrightarrow (P_c/P\ )_{\text{вх}}$.

For circuits with automatic selection, if $(P_c/P_{\text{ш}})_{\text{вх}}$ and N are set, $P_{\text{пр}}$ and $(P_c/P_{\text{ш}})_{\text{вых}}$ have unique values. Therefore, the functional relationship $P_{\text{пр}} = f\left\{(P_c/P_{\text{ш}})_{\text{вх}}\right\}$ degenerates into a point. These points for different numbers of filters N, taken from the tables given in the appendix, are designated at A_N (N = 1, 2,...,20) in Fig. 3. For a concrete value of N the inequalities (9) and (10) are fulfilled in the region (of output parameters) bounded by the coordinate axes and

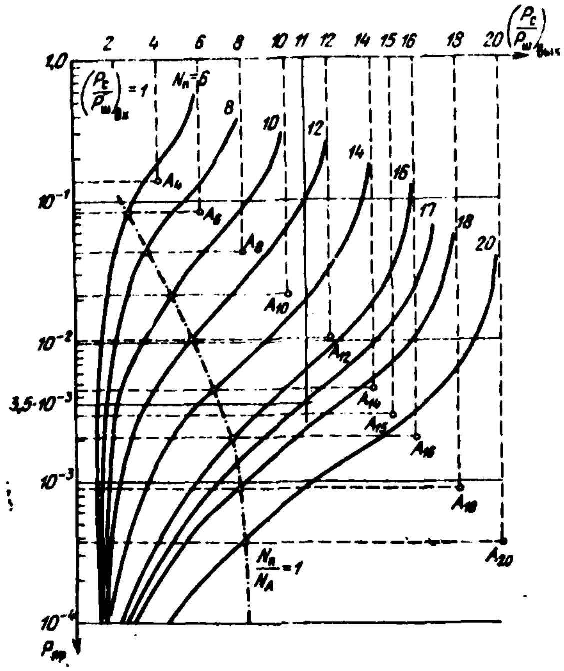

Fig. 3

the dotted lines which pass through the point A_N for circuits with automatic selection.

The number of filters required for both circuits can be readily determined from the graphs in Fig. 3. For example, let us assume that for $(P_C/P_Ш)_{вх} = 1$ we must get $(P_C/P_П)_{вых} \geqslant 11$ and $P_{пр} \leqslant 3.5 \times 10^{-3}$. We can see from the figure that the minimum number of filters in this case for a circuit with parallel connection is $N_П = 18$, and for the circuit with automatic selection it is $N = 15$.

It also follows from an analysis of the graphs that the plane for the output parameters is divided by the curve $N_{\text{п}}/N_A = 1$ (dotted striped line) into two zones for the preferred use of the circuit with automatic selection (to the right of the curve) and the preferred use of the circuit with parallel connection. A comparison of the zones shows that in the region which is most widespread in practice the values of the output parameters for the circuit with parallel connection of the filters requires a greater number of filters than the scheme with automatic selection. The line with a single point A_N, corresponds to the case in which the circuit with automatic selection has the greatest advantage. In this case the $N_{\text{п}}/N_A$ ratio reaches a value of 1.3-1.5.

Other comparative characteristics for the circuits can also be determined from Fig. 3. For example, for a set value for the probability of the signal's passage in the same number of filters we can compare the ratios $(P_c/P_{\text{ш}})_{\text{вых}}$.

APPENDIX

Table of probabilities for the passage of the signal for the automatic selection of the filters

$$10^4 \cdot P_{\text{пр}\,A} = 1 - \frac{1}{N}\sum_{i=1}^{N}(-1)^{i-1} C_N^i \, e^{-\gamma^2 (i-1/i)}$$

	$P_c/P_{\text{ш}}$											
	2	4	6	8	10	12	14	16	18	20	22	24
2	1839	676,7	248,9	91,58	33,69	12,39	4,549	1,677	0,617	0,227	0,08351	0,03072
3	2800	1122	436,8	167,1	63,14	23,67	8,824	3,277	1,214	0,4486	0,1656	0,06107
4	3440	1460	591,4	232,7	89,73	34,14	12,86	4,814	1,793	0,6656	0,2464	C,09108
5	3913	1733	724,1	291,2	114,2	43,96	16,72	6,299	2,358	0,8784	0,3261	0,1208
6	4284	1964	840,8	344,4	136,9	53,27	20,42	7,738	2,910	1,088	0,4048	0,1502
7	4588	2163	945,4	393,4	158,2	62,13	23,98	9,137	3,450	1,293	0,4822	0,1793
8	4842	2338	1040	438,8	178,4	70,61	27,42	10,50	3,979	1,496	0,5594	0,2082
9	5060	2494	1127	481,3	197,5	78,76	30,76	11,83	4,499	1,696	0,6354	0,2369
10	5250	2635	1208	521,3	215,7	86,60	34,00	13,13	5,009	1,893	0,7106	0,2653
11	5418	2764	1283	559,1	233,2	94,18	37,16	14,41	5,512	2,088	0,7851	0,2936
12	5567	2882	1353	595,0	249,9	101,5	40,23	15,65	6,006	2,280	0,8585	0,3216
13	5702	2991	1419	629,1	266,0	108,6	43,23	16,88	6,493	2,470	0,9321	0,3494
14	5824	3092	1481	661,8	285,1	115,5	46,17	18,08	6,973	2,659	1,005	0,3771
15	5935	3187	1540	693,0	296,5	122,3	49,04	19,26	7,447	2,845	1,077	0,4046
16	6038	3275	1596	723,0	311,0	128,8	51,85	20,42	7,915	3,029	1,148	0,4319
17	6133	3359	1649	751,9	325,1	135,2	54,60	21,57	8,376	3,211	1,219	0,4590

	2	4	6	8	10	12	14	16	18	20	22	24
18	6221	3437	1700	779,7	338,8	141,4	57,30	22,69	8,832	3,391	1,289	0,4860
19	6302	3512	1749	806,5	352,0	147,5	59,96	23,80	9,283	3,570	1,359	0,5128
20	6379	3582	1796	832,5	364,9	153,5	62,56	24,90	9,728	3,747	1,428	0,5394
21	6450	3649	1840	857,6	377,5	159,3	65,12	25,98	10,17	3,923	1,497	0,5660
22	6517	3713	1884	882,0	389,8	165,0	67,64	27,04	10,6	4,097	1,565	0,5923
23	6581	3775	1925	905,6	401,8	170,6	70,12	28,93	11,04	4,270	1,633	0,6186
24	6641	3833	1965	928,6	413,5	176,1	72,56	29,13	11,46	4,441	1,70	0,6447
25	6697	3889	2004	950,6	424,9	181,5	74,97	30,16	11,89	4,611	1,767	0,6707
26	6751	3943	2041	972,7	436,1	186,8	77,34	31,17	12,31	4,780	1,834	0,6965
27	6802	3994	2078	993,9	447,0	192,1	79,67	32,17	12,72	4,947	1,90	0,7223
28	6851	4044	2114	1015	457,8	197,2	81,98	33,16	13,13	5,114	1,966	0,7479
29	6897	4092	2148	1035	468,3	202,2	84,25	34,14	13,54	5,279	2,031	0,7734
30	6941	4138	2181	1054	478,6	207,2	86,49	35,11	13,94	5,442	2,096	0,7987
31	6983	4183	2213	1074	488,7	212,1	88,71	36,07	14,34	5,605	2,161	0,8240
32	7022	4226	2245	1092	498,6	216,9	90,90	37,02	14,74	5,767	2,225	0,8491
33	7060	4267	2276	1111	508,4	221,6	93,06	37,96	15,13	5,927	2,289	0,8742
34	7092	4306	2305	1129	517,9	226,3	95,19	38,89	15,53	6,087	2,353	0,8992
35	7117	4341	2335	1146	527,3	230,9	97,3	39,81	15,91	6,245	2,416	0,9240
36	7135	4370	2363	1163	536,5	235,4	99,39	40,72	16,3	6,403	2,479	0,9488

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Evaluation of the Interference Protection for Transmitting Color and Black and White Television Pictures in the FM Gain Threshold Region

E.I. Kumysh

A method is suggested for evaluating the onset of the FM gain threshold for the transmission of television pictures, and the threshold signal/noise ratios were refined for FM reception with a standard frequency detector.

The study of the interference protection for color and black and white television signals in the region of the FM threshold is of great practical importance for determining the best energy potential for the communication channel and the optimization of the electrical parameters of the VCh tracks of these channels [1]. In a widely used method for determining the FM threshold the moment for the onset of the threshold is related to a decrease in the gain in freedom from interference with respect to the potential freedom from interference, for example, at 1 dB [2]. A detailed study is almost linked with the study of the threshold curves which are used by FM demodulators. It is this approach which has predominated in the determination of the moment of onset of the FM threshold during the transmission of television signals. However, the study of the physical nature of the threshold noises and particularly their subjective perception indicates that such an approach is not adequate.

In order to facilitate the description of the following material we will give an analytical expression for the threshold curves for a standard frequency detector (ChD) for the transmission of black and white television signals. It was obtained by integrating the energy spectrum of the noise according to [3] using linear predistortions of the television signal

in the communications channel as standardized by MKKR [4]. The threshold curves were calculated as the ratio of the video-signal range from the white level to the black level to the effective, weighted voltage of the noise as a function of the ratio of the powers for the signal and the noise at the input to the frequency demodulator so that the objectively measured value of the noise will correspond to their subjective interfering action and so that the evaluation of the interference protection will correspond to that used in television [5]:

$$\frac{U_c}{U_ш}\,\text{дБ} = 9 - 10\lg\left[\frac{4}{\sqrt{3}}\,\frac{1}{m}\left(1 - \right.\right.$$
$$\left. - \operatorname{erf}\sqrt{\rho}\,\right)\int_0^1 \frac{3{,}75^2\,(1 + 14{,}7\,x^2)}{(1 + 367\,x^2)\,(1 + 155x^2)}\,dx +$$
$$\left. + \frac{1}{\rho m^2}\int_0^1 x^2\,\frac{3{,}75^2\,(1 + 14{,}7\,x^2)}{(1 + 367\,x^2)(1 + 155\,x^2)}\,dx\right]. \qquad (1)$$

Here m is the modulation index for the FM signal, ρ is the signal/noise ratio at the input to the FM receiver, $x = F/F_E$, $F_B = 6 \times 10^6$ Hz is the top frequency of the spectrum for the modulated signal, $K^2_{вст}(x) = 3{,}75^2(1+14{,}7x^2)/(1+367x^2)$ is the square of the modulus for the transmission coefficient for the restored filter; $K^2_{взв}(x) = (1+155\,x^2)^{-1}$ — квадрат --the square of the modulus for the transmission coefficient for the weighted filter.

For $K^2_{BCT}(x) = 1$, equation (1) corresponds to the case in which there is no linear treatment in the communication channel.

The results of calculations by means of equation (1) are given in Figure 1. The solid lines correspond to the direct transmission of the television signals [$K^2_{BCT}(x) = 1$] and the dotted lines correspond to the use of linear predis-

tortions in the channel. The moment for the appearance of the FM threshold, as determined by a 1 dB loss in the FM gain with respect to the potential gain, is shown by the crosses. We can see from Figure 1 that when linear predistortion is used the onset of the FM threshold is earlier (by 1.5 dB) than for the direct transmission of the television signals.

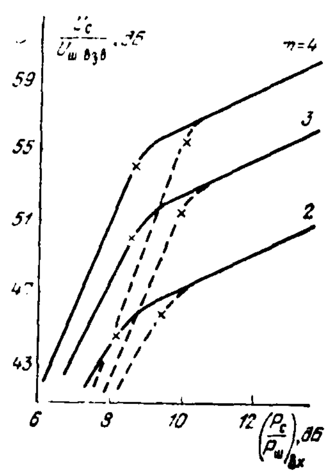

Fig. 1

However, it would be premature to consider these results to be definitive since, as we noted above, the physical nature of the threshold noises and the features for its subjective perception were not taken into account in this kind of a determination of the moment of onset for the FM threshold. In fact, in working above the FM threshold the noise at the ChD's output is a normal, steady, chance process with a zero mean [3]. Such noise covers the entire field of the picture evenly. Its interfering action can conveniently be evaluated by the value of the effective, weighted voltage which belongs to the range of the videosignal [6]. The moment for the onset of the threshold is characterized visually by the fact that individual pulses appear on the background of this noise. Their number is small at the moment the threshold appears and they approach the pulsed interference and not noise interference in their effect on the observer. If the threshold pulses at the moment the threshold sets in are identified with the pulsed interference then their interference action can be characterized best and the moment for the appearance of the FM threshold can be determined more accurately.

In television the interference from pulsed interferences is evaluated by the ratio of the ranges for the videosignal to the pulsed interference [5]. If we assume, according to [3], that the energy spectrum for the threshold pulses at the output of the FM discriminator is even then it is not difficult to show that the ratio of the videosignal range to the mean range of the pusled interference for direct transmission is equal to

$$U_c/U_n = 0{,}7\, m, \qquad (2)$$

and for linear treatment it is

$$U_c/U_n \approx 0{,}65\, m, \qquad (3)$$

where m is the modulation index for the FM signal.

Equations (2) and (3) can be used to compare the ratio of the ranges for the videosignal and the threshold interferences with those which are permissible (25 dB) for pulsed interferences as recommended by the MKKR [5]. The comparison will be made, according to [1], for the case in which the modulation index is optimum. The optimum index for the signal is selected so that a certain signal/noise ratio will be obtained at the threshold [1]. For television transmission the MKKR standards provide that this value must be equal to 55 dB. At this ratio the noise should not be noticeable in the image. It follows from Fig. 1 that the optimum modulation index for the FM signal is $m \approx 4$. However, it follows from (2) and (3) that the ratio of the ranges for the videosignal and the threshold pulses is less than the permissible value recommended by the MKKR, i.e., it is less than 25 dB. From this it may be assumed that even if the MKKR standards for the signal/noise ratio are met which, when weighted, are equal to 55 dB the threshold pulses will be clearly visible. The experiments made for

the transmission of real subjects and the test table 0249 for communications channels with FM confirmed this assumption.

These results indicate that in the threshold region it is not possible to evaluate the interference from noises by means of the relative value of the effective, weighted noise. Such an evaluation would be equivalent to the threshold pulses being unnoticeable on the image, i.e., for the condition

$$20 \lg (\dot{U}_c/\dot{U}_n) = \lg 0{,}65\, m \geqslant 25 \text{ дБ}. \qquad (4)$$

Then we get from (4) that m ≥ 27. However, if (4) is fulfilled it follows from (1) that the relative value of the effective weighted voltage for the noise greatly exceeds the MKKR standard. Therefore, it is not expedient to design communications lines with a modulation index m ≥ 27.

For m ≥ 27 and for a large enough signal/noise ratio ath the input to the demodulator, the appearance of the threshold pulse is always probable (even if it is very small) with a range for which the relationship (4) becomes invalid. However, this does not mean that m ≥ 27 should be abandoned since a frequency can be established experimentally for the appearance of threshold pulses such that they will not make the observer aware of the presence of noise in the image.

Since determining the number of threshold pulses at the moment of image transmission experimentally is complicated, the permissible signal/noise ratio at the input to the FM demodulator for which the threshold pulses will not cause the observer any awareness of noise in the image was determined by the method of subjective expertise using the method in [7]. This value for the ratio was taken as the threshold

value. The measurements were made for modulation indexes for the FM signal equal to 2, 2.5, and 3. The demodulation of the signal was accomplished with a standard ChD. The observations were made with real subjects and the 0249 test table. It was shown, as a result of the measurements, that for the direct transmission of television signals the threshold signal/noise ratio is equal to 10 dB and for linear treatment it is 10.5 dB. The somewhat greater signal/noise ratio for linear treatment as compared with the case of direct transmission of the television signals is due to the difference in the nature and subjective perception of the threshold noises. In the direct transmission of the television signals the threshold pulses appear over the entire field of the image. The maximum number of pulses occurs on black and white details for the image and fewer on the gray details. At the moment the threshold appears, for linear treatment, the pulses only appear on the borders of the sharp, high contrast differentials in brightness and as the signal/noise ratio becomes still smaller they appear on the entire field of the image.

The threshold pulses appear to be elongated along the line because of integration in the restoring filter, i.e., they have a greater duration and they become more noticeable on the screen at almost the same range [see (2) and (3)].

The data which have been presented on the value of the threshold signal/noise ratio, as well as the nature and discernibility of the threshold pulses on the image can be used to evaluate the permissible number of threshold pulses for direct transmission and for the case of linear treatment of the television signals. It was shown, as the result of the calculation, that for direct transmission the number of permissible threshold pulses is 20-25 in a single frame of the image and it is equal to 3-4 per frame for linear distortions.

In determining the protection of the transmission of color television signals from interference in communications channels with FM in the threshold region, we must, first of all, distinguish between the moment of appearance of the threshold in the communications channel and the moment of its appearance in the color channel since in a SECAM color television system the signals which differ in color are transmitted on subcarriers in the frequency band of the television signal by means of FM. For this reason the moment for the appearance of the threshold in the color channel of the SECAM system was determined by calculation and experimentally. It was found that the threshold noises in the color channel have a pulsed nature. It was shown, by using the given method with respect to the moment of appearance of the threshold, that for an even noise spectrum at the input to the color channel the moment for the appearance of the threshold is fixed for the ratio of the range of the brightness signal to the effective, weighted voltage of the noise in the brightness signal's band equal to 33-34 dB and it depends on the transmitted color. For a parabolic spectrum for the noise at the input to the color channel, which is characteristic for the direct transmission of the signal for color television in a communications channel with FM, the moment for the appearance of the threshold is fixed at a signal/noise ratio in the brightness channel equal to 45-46 dB and the threshold in the color channel, for linear predistortions in the communications channel with FM, is fixed at a signal/noise ratio in the brightness channel equal to 43-44 dB.

These results indicate that in the transmission of color television signals along communications channels with FM, which are operating in the threshold region and under the condition that the MKKR standards are met with respect to

the signal/noise ratio (55 dB) the threshold does not appear in the color channel.

It should be pointed out that the results obtained above refer to the case in which the effect of distortions of the signal for the color subcarrier SECAM system on its interference protection can be ignored.

However, when the threshold ratios in the color channel of the SECAM system and in the communications channel are similar, which may occur in practice, the effect of the distortions of the color subcarrier on its interference protection must be rigorously taken into consideration since it may cause the threshold to appear in the color channel before it appears in the communications channel.

In order to determine the moment for the appearance of the threshold in the communications channel for the transmission of color television signals, experiments were made which are analogous to the case for the transmission of black and white television signals. During the course of the experiments, with the reception of the FM signals on a standard ChD it was shown that the threshold noises begin to interfere with the visual perception of the color images almost for the same signal/noise ratio at the input to the ChD for which they begin to interfere in the transmission of black and white television signals. Therefore, all of the conclusions as to the moment for the appearance of the threshold in the communications channel with FM for the transmission of black and white television signals can also be extended to the transmission of color television signals.

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Shape of the Phase Jumps for a Narrow Band Signal

V.P. Zhukov, N.N. Ivanova, and L.A. Razumov

The results of the theoretical and experimental study of the possible shapes for the phase trajectories and the derivative for a jump in phase for the sum of a harmonic signal and a normal, narrow band noise, are presented.

The sum u(t) of the harmonic signal $E_c \cos \omega_c t$ and a normal, narrow band noise with a correlation function $\sigma^2 \rho(\tau) \cos \omega_0 \tau$ can be written in the form

$$u(t) = E(t) \cos [\omega_c t + \varphi(t)],$$

which is illustrated by the vector diagram shown in Fig. 1.

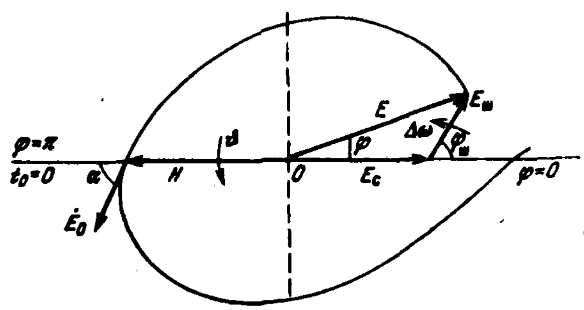

Fig. 1

In the presence of detuning $\Delta\omega=\omega_0-\omega_c$ between the frequency of the harmonic signal and the central frequency of the noise spectrum, the noise vector not only fluctuates in amplitude

and phase but it also rotates in the plane of the diagram with respect to the end of the E_c vector at an angular velocity $\Delta\omega$. Large fluctuations in $E_ш$ and $\varphi_ш$ can result in the complete revolution of the vector E around the point 0, i.e., to a 2π phase jump. The phase's trajectory $\varphi(t)$ for a jump is chance and, as will be shown below, its form depends on the detuning $\Delta\omega$ and on the magnitude and rate of fluctuation of the envelope E(t) and of the phase $\varphi(t)$ which accompany the given jump.

Let $t_0 = 0$ be the phase at some moment of time and its derivative will assume the value $\varphi_0 = \pi$, $\varphi'_0 = \vartheta$ and the envelope and its derivative have the values $E_0 = H$ and $E_0' = \gamma = H\vartheta/\mathrm{tg}\,\alpha$ (α is the angle of slope for the trajectory of the end of the vector E to the abscissa $\varphi_0 = \pi$).

Then the behavior of the phase $\varphi(t)$ and of the envelope E(t) at $\infty \leqslant t < \infty$ is described by the conditional probability density $w_{усл}(E, \varphi)$ which can be calculated by the formula

$$w_{усл}(E, \varphi) = \left.= \frac{w(E_0, \varphi_0, E_0', \varphi_0', E, \varphi)}{w(E_0, \varphi_0, E_0', \varphi_0')}\right|_{\varphi_0=\pi,\ E_0=H,\ E_0'=\gamma,\ \varphi_0'=\vartheta.} \qquad (1)$$

The denominator in equation (1) is the four dimensional probability density for the quantities φ_0, E_0, E_0', and φ_0', at the moment of time t_0 and the numerator is the six dimensional probability density for these same quantities and for the envelope E and the phase φ, at the moment of time t. The method for calculating these probability densities is well known [2].

Mathematical transformations which are not complex but are cumbersome, made according to equation (1) give

$$w_{\text{усл}}\,[E(t),\ \varphi(t)] = \frac{E(t)}{2\pi\sigma^2(1-\dot{\rho}^2/\delta\omega^2-\rho^2)} \times$$

$$\times \exp\left\{\frac{E^2(t)-2E(t)A\cos[\varphi(t)-\Phi]+A^2}{-2\sigma^2(1-\dot{\rho}^2/\delta\omega^2-\rho^2)}\right\}, \qquad (2)$$

in which $\delta\omega^2=-\ddot{\rho}(0)$ A and Φ are functions of time which depend on $\Delta\omega$, the correlation coefficient of the noise $\rho(\tau)$, the quantities E and φ and their derivatives for $t_0 = 0$ and are determined by the formulas

$$A^2 = (E_c+E_1)^2+E_2^2,$$

$$\Phi = \operatorname{arctg}\frac{E_2}{E_c+E_1}, \qquad (3)$$

in which

$$E_1 = x\cos\Delta\omega t - y\sin\Delta\omega t,\quad E_2 = x\sin\Delta\omega t + y\cos\Delta\omega t,$$
$$x = -\rho(t)(E_c+H)+\dot{\rho}(t)\gamma/\delta\omega^2,\quad y=\dot{\rho}(t)[H\vartheta -$$
$$-\Delta\omega(E_c+H)]/\delta\omega^2.$$

We can see from an analysis of (2) that the function A(t) characterizes the behavior of the envelope E(t) in the jump and the function Φ(t) is the most probable value for the phase φ(t).

The purpose of this work was to study the phase trajectories, the most likely one being described by the expression

$$\Phi(t) = \operatorname{arctg}\frac{-[(1+h)R+NkR_1]\sin\Delta\omega t - R_1[N-}{1-[(1+h)R+kNR_1]\cos\Delta\omega t+R_1[N-} \rightarrow$$

$$\rightarrow \frac{-f(1+h)]\cos\Delta\omega t}{-f(1+h)]\sin\Delta\omega t}. \qquad (4)$$

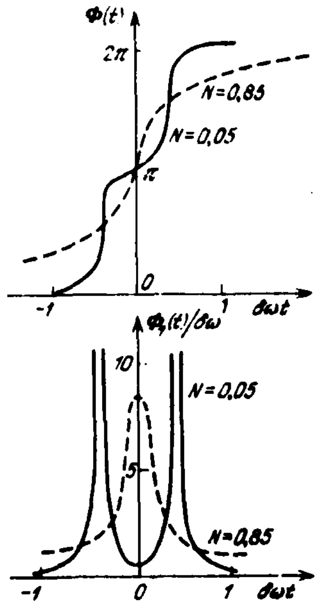

Fig. 2

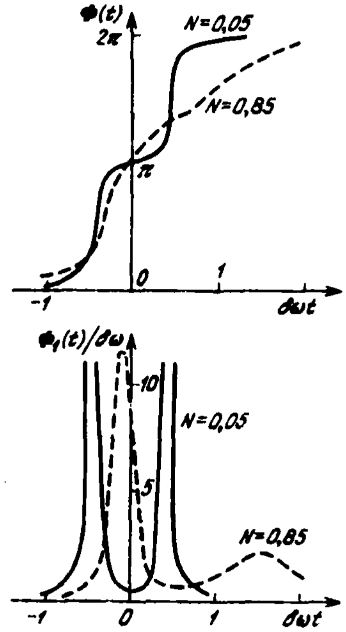

Fig. 3

The following designations have been used: $\rho(t) = R$, $-\dot{\rho}(t)/\delta\omega = R_1$, $\Delta\omega/\delta\omega = f$, $H/E_c = h$, $h\vartheta/\delta\omega = N$, $1/\mathrm{tg}\,\alpha = k$.

Differentiating $\Phi(t)$ with respect to time gives the derivative of the most probable phase trajectory $\Phi_1(t) = d\Phi(t)/dt$, which characterizes the rate of change in the phase in the jump.

Some of the results of the calculation of the most probable phase trajectories for $\Phi(t)$ and its derivative $\Phi_1(t)$ for the case in which $\rho(t) = \exp(-\alpha^2 t^2/2)$ are shown in Figs. (2-6).

Figures 2 and 3 illustrate the effect of the parameter N = $E_0\varphi_0'/E_c\delta\omega$ on the shape of the jumps for zero detuning $f = \Delta\omega/\delta\omega$ and h - 0.1. The trajectories shown in Fig. 2, were set up for k = 0 and in Fig. 3 for k = 1. An analysis of the graphs shows that the deviation for the parameter k from zero leads to an asymmetric trajectory. We note that the value of the parameter N has little effect on the duration of the phase jump.

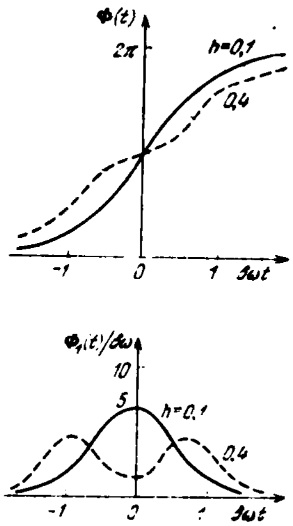

Fig. 4

Two trajectories are shown in Fig. 4 (for k = 0, N = 0.5, and f = 0) which differ in the value of $E_0 = hE_c$. For small values of the envelope, the jumps are smoother and shorter in duration.

A group of trajectories is shown in Fig. 5 (set up for k = 0, N = 0.5, and h = 0.5) which correspond to a different frequency difference $\Delta\omega = f\delta\omega$. As we might expect, the negative detuning hinders a positive jump. For large negative detunings the negative jumps are most probable.

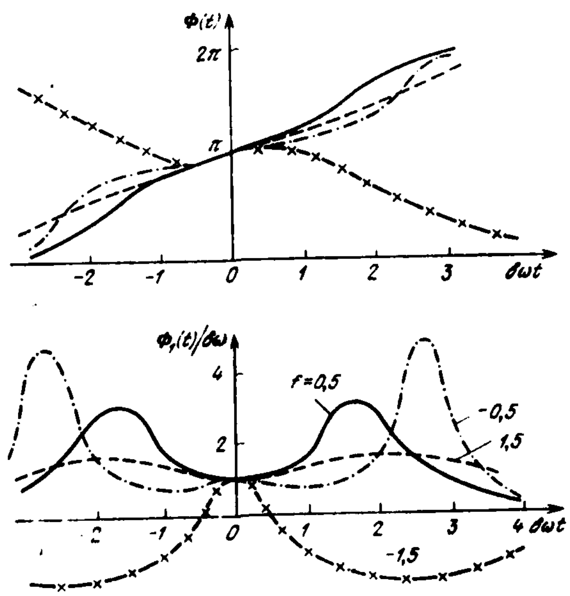

Fig. 5

The experimental study of the phase trajectories during jumps was carried out by the method described in [1]. As an example, the experimental trajectories (dotted and strip-dot lines) are shown in Fig. 6 along with the trajectories calculated by using equation (4) (solid lines). The experimental results confirm the fact that the curves, calculated by means of equation (4) are, in fact, the most probable trajectories for the phase during the jump.

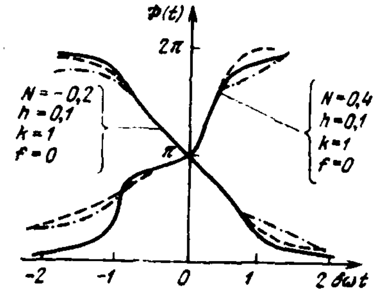

Fig. 6

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Mutual Correlation of the Pulsed and Gaussian Components of the Noise at the Output to the Frequency Discriminator

N.A. Rabinovich

A statistical analysis is given of the switching model for noise at the output of the frequency discriminator (ChD). Expressions are found for the correlation function and the energy spectrum for the noise at the ChD's output for a modulated signal and an additive, Gaussian noise at its input.

The most widespread statistical models for noise today [1, 2] can be used to describe the energy spectrum at the ChD's output only in the vicinity of the threshold and above it. The experimental values for the energy spectrum for the noise at the output to the ChD for the deep, subthreshold region of an input signal/noise ratio ρ are in poor agreement with the theoretical results (particularly for a modulated signal) found from the S.O. Rice pulse model and the method of weighted probabilities.

In this article an idea is presented on the possibility of representing noise at the ChD's output in the form of the output signal from a composite source (so-called switching model) for noise).

The purpose of this work is the statistical analysis of the switch model for noise which is made for the assumption that a signal, which is frequency modulated, and an additive, Gaussian noise act on the input of an ideal ChD. The correlation functions and the energy spectrum for the noise at the ChD's output are found. Particular attention is given to the mutual correlation which occurs between the pulsed

and Gaussian components of the noise at the ChD's output.

Let us represent the noise u(t) at the ChD's output in the form of the sum of two components, the Gaussian $\eta(t)$ and the pulsed p(t)

$$u(t) = \eta(t) + p(t) = a(t)\,\eta_0(t) + [1 - a(t)]\,p_0(t), \quad (1)$$

where $\eta_0(t)$ and $p_0(t)$ are the generating processes for the Gaussian and pulsed components of the noise, respectively and a(t) is the switching process which is independent of them and which assumes values of 1 and 0 when the corresponding Gaussian and pulsed components of the noise act on the ChD's output.

The generating process for the Gaussian component of the noise is of course assumed to be a chance process:

$$\eta_0(t) = y'(t)/Q, \quad (2)$$

where y(t) is the quadratic component of the input noise and Q is the amplitude of the frequency modulated signal. It follows from (2) that the generating Gaussian process is characteristic for the operation of a ChD for $\rho \gg 1$.

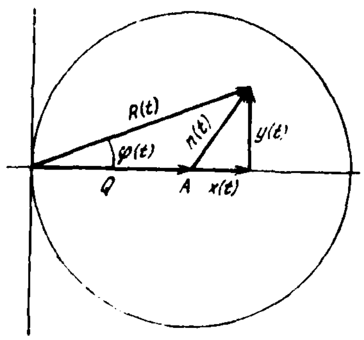

Fig. 1

We will assume that the Gaussian component for the noise at the ChD's output acts in that time segment for which the vector for the sum of the received signal and noise falls into a circle or radius Q with its center at the point A (Fig. 1). We can readily show that in this case the mean value a_0 for the

switching process a(t) is equal to

$$a_0(\rho) = 1 - e^{-\rho}. \qquad (3)$$

The correlation function for the noise u(t) can be written in the form

$$R_u(t) = R_p(\tau) + 2R_{\eta p}(\tau) + R_\eta(\tau), \qquad (4)$$

where $R_p(\tau)$ and $R_\eta(\tau)$ are the correlation functions for the pulsed and Gaussian components of the noise and $R_{\eta p}(\tau)$ is their mutual correlation function.

We have the following expression for the correlation function for the Gaussian component of the noise

$$R_\eta(\tau) = \overline{a(t)\,a(t+\tau)}\;\overline{\eta_0(t)\,\eta_0(t+\tau)} = R_a(\tau)\frac{g''(\tau)}{Q^2}, \qquad (5)$$

where $R_a(\tau) = \overline{a(t)\,a(t+\tau)}$ is the correlation function for the process a(t); $g(\tau)$ is the correlation function for y(t).

The mutual correlation function $R_{\eta p}(\tau)$ is given in the form

$$R_{\eta p}(\tau) = \overline{[u(t) - a(t)\,\eta_0(t)]\,a(t+\tau)\,\eta_0(t+\tau)} =$$
$$= R_a(\tau)\,g''(\tau)/Q^2 + a_0\,\overline{u(t)\,\eta_0(t+\tau)}. \qquad (6)$$

In equation (6) there is a term which represents the mutual correlation between the Gaussian generation process $\eta_0(t)$ and the noise u(t) (at the ChD's output):

$$\overline{u(t)\,\eta_0(t+\tau)} =$$
$$= \overline{\frac{[x(t) + Q]\,y'(t) - y(t)\,x'(t)}{[x(t) + Q]^2 + y^2(t)}\;\frac{y'(t+\tau)}{Q}}. \qquad (7)$$

Calculation of the latter expression gives [3]:

$$\overline{u(t)\,\eta_0(t+\tau)} = -\frac{g''(\tau)}{Q}(1-e^{-\rho}) - \frac{b_1 h'(\tau)}{b_0^2}e^{-\rho}, \quad (8)$$

$$b_n = (2\pi)^n \int_0^\infty S_{вх}(f)(f-f_c)^n\,df,$$

where
$$g(t) = \int_0^\infty S_{вх}(f)\cos 2\pi(f-f_c)\tau\,df;$$

$$h(\tau) = \int_0^\infty S_{вх}(f)\sin 2\pi(f-f_c)\,d\tau f,$$

where $S_{вх}(f)$ is the energy spectrum for the input noise, f_c is the detuning frequency for the sum of the signal and the noise with respect to the symmetry frequency of the input filter.

An important moment in the analysis of the switching model for noise is in defining the form of the correlation function for the process a(t). In the general case this problem is quite complicated however approximate methods can be considered for finding $R_a(\tau)$. We will pause here on two such methods.

1. We will assume that the energy spectrum for the process a(t) is centered in the range of low frequencies. In this case the correlation function $R_a(\tau)$ can be given in the form

$$R_a(\tau) \approx R_a(0) = 1 - e^{-\rho}. \quad (9)$$

This expression follows from the fact that the values of the process a(t) can only be 0 and 1 and that its mean value is determined by the formula (3).

By substituting (9) in (5) and (6) and by using (8) we get the correlation function $R_u(\tau)$ in the form

$$R_u(\tau) = R_p(\tau) - (1-e^{-\rho}) \times$$
$$\times (1-2e^{-\rho})g''(\tau)/Q^2 - 2(1-e^{-\rho})e^{-\rho} b_1 h'(\tau)/b_0^2. \quad (10)$$

The energy spectrum $S_u(f)$ for the noise at the ChD's output can be found by carrying out the Fourier conversion for $R_u(\tau)$:

$$S_u(f) = 4\pi^2 (N_+ + N_-) +$$
$$+ \frac{(1 - e^{-\rho})(1 - 2e^{-\rho})}{2\rho} (2\pi f)^2 S_{\text{вх}}(f + f_c) -$$
$$- 2(2\pi)^2 (1 - e^{-\rho}) e^{-\rho} f b_1 S_{\text{вх}}(f + f_c),$$
$$-\infty < f < \infty, \qquad (11)$$

where $N_{\pm}$ is the mean number of phase clicks for the sum of the input signal and the noise $\pm 2\pi$, respectively. In (11) we used Rice's approximation for the energy spectrum of the pulsed component of the noise [2]. It follows from (11) that in addition to an even spectrum for the pulsed component of the noise and a parabolic spectrum, which is characteristic for the operation of the ChD above the threshold, there is a term with a linear frequency function which arises because of the mutual correlation of the pulsed and Gaussian components of the output noise. It should be pointed out that this term is different than zero only for a modulated signal ($b_1 \neq 0$).

2. Let us write the process a(t) in the form

$$a(t) = \tilde{a}(t) + a_0,$$

where $\tilde{a}(t)$ is the chance process with a zero mean. The correlation function $R_a(\tau)$ is equal to

$$R_a(\tau) = R_{\tilde{a}}(\tau) + a_0^2,$$

where $R_{\tilde{a}}(\tau)$ is the correlation function for the process $\tilde{a}(t)$. For large signal/noise ratios ($\rho > 1$)

$$R_a(\tau) \approx a_0^2. \qquad (12)$$

By using (12) the correlation function for the noise at the ChD's output is given in the form

$$R_u(\tau) = R_p(\tau) - (1 - e^{-\rho})^2 g''(\tau)/Q^2 -$$
$$- 2(1 - e^{-\rho}) e^{-\rho} b_1 h'(\tau)/b_0^2. \qquad (13)$$

The following expression is arrived at for the energy spectrum of the noise at the ChD's output from (13)

$$S_u(f) = 4\pi^2 (N_+ + N_-) + \frac{(1 - e^{-\rho})}{2\rho} (2\pi f)^2 \times$$
$$\times S_{\text{вх}}(f + f_c)$$
$$- 2(2\pi)^2 (1 - e^{-\rho}) e^{-\rho} f b_1 S_{\text{вх}}(f + f_c),$$
$$-\infty < f < \infty. \qquad (14)$$

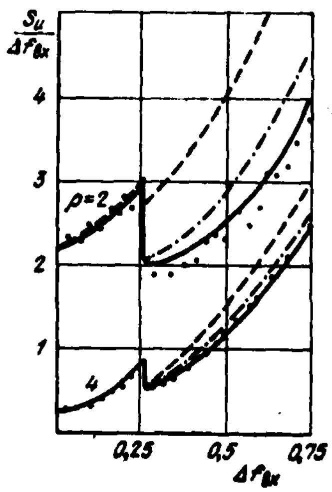

Fig. 2.

A comparison of the theoretical and experimental results is shown in Fig. 2. Here the experimental results for the energy spectrum of the noise at the ChD's output are shown by the dots for a constant detuning of the signal with respect to the symmetry frequency of the input filter with a rectangular frequency characteristic ($\Omega_0 = \Delta f/\Delta f_{\text{вх}} = 0{,}25$). The dotted curve corresponds to the spectrum given by S.O. Rice's puls model [1, 2] (taking into accoun the suppression of the Gaussian component of the noise) and the solid curve was calculated by using equation (11) and the stripe-dot curve was calculated by means of equation (14). It follows from Fig. 2 that for the deep, subthreshold region ($\rho \approx 1$), the best approximation of the experimental results is given by equation (11).

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The Concept of Fluctuation Interference in Reduced Coordinate Systems

V.A. Zaytsev and V.I. Menenkov

The concept of chance processes in reduced coordinate systems is discussed in which the intrinsic oscillation for the equations for nonstationary systems are described as the "simplest" oscillations. A foundation is given for the concepts of correlation function, energy spectrum, transmission coefficient, white noise, and reduced normalized coordinates.

The problem of observing the filtering complex signals is simplified if the area of observation is represented in a basic system of intrinsic functions by equations for the filtering and generating circuits. However, a comparative analysis for different communications systems should be made by using one of the coordinate systems for the functions. The concept of reduced scales for time and amplitude [1] can be used to reduce the different basic systems, which are adequate for nonstationary filters, to a single, more convenient system, for example, a basic system of functions which is used in the classical Fourier transformation.

In this study, a basis is given for models of chance processes which, in the basic system of intrinsic functions for a nonstationary circuit, correspond to the stationary processes for circuits with constant parameters.

Let us consider functions having the form

$$s(t) = \lambda(t)\, e^{j\omega\tau(t)}, \qquad (1)$$

which are intrinsic functions for the structural-signal parametric filters (SSPF) [2]. In the reduced time scale

$\tilde{t} = \tau(t)$ and the normalized amplitude scale (normalizing factor $1/\lambda(t)$, the functions (1) describe monoharmonic oscillations

$$s(\tilde{t}) = e^{j\omega\tilde{t}}. \qquad (2)$$

A group of functions of the type (2) are chosen as the basis for the canonical representation of chance processes

$$s_i(\tilde{t}) = e^{j\omega_i\tilde{t}}, \qquad (3)$$

which are orthogonal in some range $[0, \tilde{T}]$ of the reduced time scale. The convolution of the inverted function x(t) in this range (realization of the chance process) with a function from the group in (3) is

$$x_i = \int_0^{\tilde{T}} x(t)\, s_i(\tilde{t})\, dt$$

a unique dependence $t(\tau)$ determines the projection of the realization on the coordinate function $s_i(\tilde{t})$ uniquely.

The condition for the completeness for the base of the functions in (3) in converting from the time for the complete orthonogal basis for functions of the type $f_i = \exp j\omega_i t$ which is known in the real time scale, by replacing the variable t by $\tilde{t}$ is determined by the uniqueness of the function $\tau(t)$ and its inverse function $t(\tau)$ is an example we will represent an element of the initial base $z_i(t)$ by the sum

$$z_i(t) = \sum_{j=0}^{\infty} c_j s_j(\tilde{t}),$$

where $s_j(\tilde{t})$ is an element of the base obtained by replacing the variable which has be orthonormalized in the reduced coordinates. The completeness of the new basis can be proved from the Parseval equation

$$\int_0^{\tau_T} z_i^2 [t(\ )]\, dt = \sum_{j=0}^{\infty} c_j^2;\ \tau_T = \tau(T).$$

In the real time scale the elements of the base $\{s_i(\tilde{t})\}$ are orthogonal with a weight of $\dot{\tau}(t)$ which is different, usually, from unity, which limits the space of the function which can be represented in the base in (3) by the condition

$$\int_0^T x^2(t)\, \dot{\tau}(t)\, dt < \infty.$$

For a comparative analysis of the freedom from interference for filters with variable and constant parameters by the method of reduction of the intrinsic functions of the different circuits to a single, base system of functions energy relationships are needed in the initial and reduced scales for measuring the corresponding variables.

Let us consider a chance process, which is stationary in the reduced coordinates and which has an even energy spectrum. The autocorrelation function for such a process, which we will call corrected white noise [1] is determined by the relationship $R(\Delta\tilde{t}) = N_0\delta(\Delta\tilde{t});\ \Delta\tilde{t} = \tau(t_1) - \tau(t_2)$.

It is not difficult to show, by taking (1) and (2) into account that in real coordinates this function has the form

$$R(t_1, t_2) = N_0\, \ddot{\lambda}(t_1)\, \lambda(t_2)\, \delta[\tau(t_1) - \tau(t_2)]. \qquad (4)$$

The conversion of the chance process $x(t)$ with a non-stationary filter can be written in the form of the integral [3]

$$u(t) = \int_{-\infty}^{\infty} g(t, \theta)\, x(\theta)\, d\theta,$$

where g(t, θ) is the filter's pulsed characteristic.

Let us find the pulsed intermediate characteristic for the SSPF [2]. The following property of the δ-function is known [3]:

$$\delta[\tau(t)] = \sum_i \frac{\delta_*(t_* - t_i)}{|\dot{\tau}(t)|},$$

where t_i is the simple root of the equation $\tau(t) = 0$.

From the conditions for the ability for the physical realization of the nonstationary system (1) it follows that $\dot{\tau}(t) \geq 0$ (conditions of stability) therefore the equation $\tau(t) = 0$ has only one root, $t_i = 0$. It follows from this that the delta functions in the real and reduced time are related by the equation $\delta(t) = \dot{\tau}(t)\,\delta[\tau(t)]$.

By taking the normalizing factor for the amplitude scale as equal to $1/\lambda(t)$ we get, in the reduced coordinates

$$\delta\,(\tilde{t}) = \dot{\tau}(t)\,\delta\,[\tau(t)]/\lambda(t).$$

Now it is not difficult to show that the expression for the pulsed characteristic of the SSPF can be written in the form

$$g(t, x) = \lambda(x)\,\frac{\dot{\tau}(t)}{\lambda(t)}\,g_0[\tau(x) - \tau(t)],$$

in which g_0 is the pulsed characteristic for a system with constant parameters to which the SSPF is reduced by replacing the variable.

By making use of the conclusions in [3] we can find the relationship between the correlation functions at the input and output of the SSPF:

$$R_2(t_1, t_2) = \lambda(t_1)\lambda(t_2)\iint_{-\infty}^{\infty} R_1(\theta_1, \theta_2) \times$$

$$\times \frac{\dot{\tau}(\theta_1)\dot{\tau}(\theta_2)}{\lambda(\theta_1)\lambda(\theta_2)} g_0[\tau(t_1) - \tau(\theta_1)]\, g_0[\tau(t_2) -$$

$$- \tau(\theta_2)]\, d\theta_1\, d\theta_2.$$

We will introduce the designation $R(\tilde{t}_1, \tilde{t}_2) = R(t_1, t_2)/\lambda(t_1)\lambda(t_2)$. Then in the reduced time for the normalized amplitude scale we get

$$R_2(\tilde{t}_1, \tilde{t}_2) = \iint_{-\infty}^{\infty} R_1(\tilde{\theta}_1, \tilde{\theta}_2)\, g(\tilde{t}_1 - \tilde{\theta}_1)\, g(\tilde{t}_2 - \tilde{\theta}_2)\, d\tilde{\theta}_1\, d\tilde{\theta}_2.$$

If the input process is stationary in the reduced coordinates $[R_1(\tilde{t}_1, \tilde{t}_2) = R_1(\tilde{t}_1 - \tilde{t}_2)]$, then the function for the autocorrelation at the output for the SSPF is determined by the relationship

$$R_2(\Delta\tilde{t}) = \iint_{-\infty}^{\infty} R_1(u - v + \Delta\tilde{t})\, g(u)\, g(v)\, du\, dv,$$

where $u = \tilde{t}_1 - \tilde{\theta}_1$ and $v = \tilde{t}_2 - \tilde{\theta}_2$.

We will show that the generalized Viner-Khinchin theory [4] for SSPF relates the corrected energy spectrum and the corrected correlation function by the Fourier transformation in the corrected coordinates. For a truncated realization of the process $n^{(K)}(t)$ equal to zero beyond the limits of the [-T/2, T/2] range the generalized Fourier transformation is valid

$$\gamma_i^{(k)}(\omega) = \int_{-T/2}^{T/2} n^{(k)}(t)\, \frac{\dot{\tau}(t)}{\lambda(t)}\, e^{-j\omega\tau(t)}\, dt.$$

The mean power of the process at a frequency ω, with reference to the band $\Delta f = 1/T$, is equal to

$$W_k^{(h)}(\omega) = \frac{2}{T} - |\eta^{(k)}(\omega)|^2 = \frac{2}{T} \iint_{-T/2}^{T/2} n_1(t_1) \times$$

$$\times n_2(t_2) \frac{\dot{\tau}(t_1)\,\dot{\tau}(t_2)}{\lambda(t_1)\,\lambda(t_2)} \exp\{-j\omega_k^t[\tau(t_1) - \tau(t_2)]\}\, dt_1\, dt_2.$$

By averaging this expression over multiple realizations we find

$$W(\omega) = \iint_{-T/2}^{T/2} R(t_1, t_2) \frac{\dot{\tau}(t_1)\,\dot{\tau}(t_2)}{\lambda(t_1)\,\lambda(t_2)} \times$$

$$\times \exp\{-j\omega[\tau(t_1) - \tau(t_2)]\}\, dt_1\, dt_2. \qquad (5)$$

For a chance process which is stationary in the reduced time scale and whose correlation function is written in the form

$$R(t_1, t_2) = \lambda(t_1)\,\lambda(t_2)\, R[\tau(t_1) - \tau(t_2)], \qquad (6)$$

expression (5) can be given as

$$W(\omega) = \frac{2}{T} \iint R(\tilde{t}_1 - \tilde{t}_2)\, e^{-j\omega(\tilde{t}_1 - \tilde{t}_2)} d\tilde{t}_1\, d\tilde{t}_2.$$

Let us introduce a new variable, $z = \tilde{t}_1 - \tilde{t}_2$. By dividing the area of integration along the diagonal $\tilde{t}_1 = \tilde{t}_2$ we get

$$W(\omega) = 2 \int_{-T}^{T} \left(1 - \frac{|z|}{T}\right) R(z)\, e^{-j\omega z}\, dz.$$

for $T \to \infty$

$$W(\omega) = 2 \int_{-\infty}^{\infty} B(\tilde{t})\, e^{-j\omega\tilde{t}}\, d\tilde{t}.$$

Now let us find the corrected spectrum for the process at the output of the nonstationary filter

$$W(\omega) = 2\int_{-\infty}^{\infty} B(\tau)\, e^{-j\omega\tau}\, d\tau =$$

$$= \int_{-\infty}^{\infty} g_0(u)\, g_0(v) \int_{-\infty}^{\infty} R_s(u - v + \tau)\, e^{-j\omega\tau}\, d\tau\, du\, dv =$$

$$= \int_{-\infty}^{\infty} g_0(u)\, e^{j\omega u}\, du \int_{-\infty}^{\infty} g_0(v)\, e^{-j\omega v}\, dv \cdot 2 \int_{-\infty}^{\infty} R_s(\tau) \times$$

$$\times\, e^{-j\omega\tau}\, d\tau = k^2(\omega) F(j\omega), \qquad (7)$$

here $k(j\omega)$ is the reduced transmission coefficient of the SSPF, $F(j\omega)$ is the corrected energy spectrum for the chance process at the filter's output.

The mean power for the process, which, from (6) is equal to

$$P = \frac{1}{2\pi} \lambda^2(t) \int_{-\infty}^{\infty} W(\omega)\, d\omega,$$

is a function of time. Averaging with respect to time gives

$$P = \frac{1}{2\pi} \int_{-\infty}^{\infty} W(\omega)\, d\omega \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \lambda^2(t)\, dt.$$

The following expression is obtained for the mean power for the process at the output of the SSPF taking (7) into account

$$P_{\text{вых}} = \frac{1}{2\pi} \lambda^2_{\text{ср}} \int_{-\infty}^{\infty} k^2(\omega)\, F(\omega)\, d\omega.$$

If λ = const, then the results of calculating the energy relationships by the corrected methods for the passage of chance processes through the FM SSPF coincide with the analogous results for filters with constant parameters. In the general case of AM-FM SSPF it is necessary in calculations of the

freedom from interference to introduce the correction coefficient

$$\lambda^2_{cp} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \lambda^2(t)\,dt.$$

The average power of the interference at the output to the filter for the action of corrected white noise on the SSPF, when $F(\omega)$ = const, is given by the quantity

$$P_s = \frac{1}{2\pi}\lambda^2_{cp}\int_{-\infty}^{\infty} k^2(\omega)\,d\omega.$$

A study of the signals of the type of (2) does not limit the general nature of the concepts presented here which can be extended, for example, to signals of the type

$$a(t) = A_1 a_1(t) + A_2 a_2(t). \qquad (8)$$

By using the designations $s = a_i(t) + ja_2(t)$ and $s^* = a_1(t) - ja_2(t)$, we can write

$$s = \lambda(t)\,e^{j\psi(t)} = \lambda(t)\,e^{j\omega_s(t)};\; s^* = \lambda(t)\,e^{-j\psi(t)},$$

in which the modulus is

$$\lambda(t) = \sqrt{A_1^2 a_1^2(t) + A_2^2 a_2^2(t)},$$

and the argument is

$$\psi(t) = \operatorname{arctg}\frac{a_1(t)}{a_2(t)} = \frac{1}{2j}\ln\frac{s}{s^*}.$$

By using these designations the correlation function, according to (4) for corrected white noise can be written in the form

$$R(t_1, t_2) = N_0 \sqrt{s(t_1) s^*(t_1) s(t_2) s^*(t_2)} \times$$
$$\times \delta \left\{ \frac{1}{2j} \left[\ln \frac{s(t_1)}{s^*(t_1)} - \ln \frac{s(t_2)}{s^*(t_2)} \right] \right\} =$$
$$= N_0 \sqrt{s(t_1) s^*(t_1) s(t_2) s^*(t_2)} \times$$
$$\times \delta \left[\frac{1}{2j} \ln \frac{s(t_1) s^*(t_2)}{s^*(t_1) s(t_2)} \right].$$

Accordingly the correction coefficient is

$$\lambda_{cp}^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T s(t) s^*(t) dt.$$

Thus, the introduction of the concept of reduced chance processes simplifies the calculation of the freedom from interference for nonstationary systems, reducing it to methods of calculation which have been well devleoped for the freedom from interference for circuits with constant parameters and, in addition, the comparison of the freedom from interference for circuits with constant and variable parameters is simplified.

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The Use of Modulated Filters for FM and AM Reception On a Background of White Noise

I.D. Zolotarev and S.V. Bukharin

A number of problems for the interference free reception have been successfully solved by using modulated filters [1], and the use of a filter with a variable transmission band width decreases the duration of the transition process and increases the signal/noise ratio at the output of the system.

In this work the passage of a mixture of FM or AM signals with white noise through a filter with a variable band width was studied. A circuit, which is shunted by a period of time τ after manipulation of the radio signal's parameters (the moment of manipulation is assumed to be the beginning of the reading) is used for this purpose. In this case the problem for the passage of an FM signal is reduced to the problem of switching on a radio pulse in a circuit with nonzero starting conditions due to the energy which is stored during the passage of the preceding elementary signal pulses.

Let us consider the range [0, T] in which the radio signal's parameters (duration of the AM signal or the discreteness of the FM signal) are constant. The initial equation for determining the component U(t) at the filter's output has the form

$$\frac{d^2U}{dt^2} + 2\alpha(t)\frac{dU}{dt} + \omega_p^2 U = E \sin(\omega_H t + \psi), \qquad (1)$$

where $\alpha(t) = 1/2R(t)C$ and $\alpha(t)$ changes according to the law

$$\alpha(t) = \begin{cases} k\alpha, & 0 \leqslant t < \tau; \\ \alpha, & \tau \leqslant t < T \end{cases} \quad k > 1. \qquad (2)$$

In the $[0,\tau]$ range (the case of a "wide" band) the response equation is found by the Laplace transformation taking into account the initial conditions U(0) and U'(0):

$$\bar{U}(t) = \frac{E[(-k\alpha + j\omega_0)\sin\psi + \omega_H\cos\psi]}{j\omega_0[-k\alpha + j(\omega_0 - \omega_H)][-k\alpha + j(\omega_0 + \omega_H)]} \times$$
$$\times e^{(-k\alpha + j\omega_0)t} + \frac{E(j\omega_H\sin\psi + \omega_H\cos\psi)}{j\omega_H[k\alpha + j(\omega_H - \omega_0)][k\alpha + j(\omega_H + \omega_0)]} \times$$
$$\times e^{j\omega_H t} + \frac{\bar{U}'(0) + (k\alpha + j\omega_0)\bar{U}(0)}{2j\omega_0} e^{(-k\alpha + j\omega_0)t}. \qquad (3)$$

For $\omega_0 = \omega_H$ and $\alpha(t) \ll \omega_0$, the value of the derivative at any moment of time is related with the value of the signal

$$\bar{U}'(t) = j\omega_0\bar{U}(t). \qquad (4)$$

The initial conditions $U(\tau)$ and $U'(\tau)$ for the range $[\tau,T]$ (the case of a "narrow" band) is determined by taking (3) and (4) into account. In this range the solution is determined by equation (3) for k = 1, for nonzero starting conditions at the moment τ, and by replacing ψ by $\psi_\tau = \psi + \omega_H\tau$, t by $t_1 = t - \tau$. The system's reaction to the action of an AM signal is also determined by equation (3). The difference lies in the fact that the initial conditions are zero conditions (t = 0). The results that were obtained for a manipulated parameter are extended to the arbitrary law for the "slow" change in the parameters by means of the concept of conditional, generalized functions which were developed for modulated filters [1].

Taking into account the expression for the dispersion of the chance component at the output of the circuit for the action of white sound on it [2], we get, for the interval $[0, \tau]$

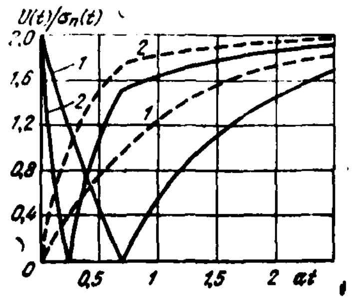
$$\sigma_n^2(t) = \frac{N_0 R}{4C}\left[\frac{k_0}{k} + \left(1-\frac{k_0}{k}\right)e^{-2k\alpha t}\right], \qquad (5)$$

in which k_0 is the coefficient for the change in the amplification.

In the interval $[\tau, T]$ the dispersion for the chance component is

$$\sigma_n^2(t_1) = \frac{N_0 R}{4C}\left\{1+\left[\left(\frac{k_0}{k}-1\right)-\left(\frac{k_0}{k}-1\right)e^{-2k\alpha\tau}\right]e^{-2\alpha t_1}\right\}. \qquad (6)$$

The curves which characterize the signal/noise ratio at the filter's output are shown in the figure (solid lines for FM and dotted lines for AM signals). Here curves I are set up for a stationary filter, curves 2 are for a nonstationary filter with $k = 3$, $k_0 = 3$, $\alpha\tau = 0.7$. For a given ratio of the established value of the signal to the effective noise level $U_{ycr}/\sigma_{n\,ycr}$ the FM signal at the output exceeds the noise for the stationary system at a time equal to $1.37\alpha t$ and at a time equal to $0.45\alpha t$ for a nonstationary system. The average signal/noise ratio during the period of treatment



$$I = \frac{1}{T-\tau}\int_{\tau}^{T}\frac{U(t)}{\sigma_n(t)}\,dt \qquad (7)$$

is equal to 1.098 for the stationary filter and 1.764 for the nonstationary filter.

The results lead to the conclusion that a modulated filter is better than a stationary filter for FM and AM reception both with respect to the criterion of a minimal time in which the signal is a given range higher than the mean square level for the noise and in terms of the mean signal/noise ratio.

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Study of the Operation of a Dynamic Discriminator of the Outside Frequency for the Signal's Spectrum in the Presence of Fluctuation Noises

N.S. Shlyakov

A fluctuation circuit is studied for the discriminator of the outside frequency for the signal's spectrum. Its stability is studied by an approximate method "on a small scale." The results are given of a study of the discriminator's operation under conditions of dynamic tracking of the outside frequency of the signal's spectrum on a background of interfering signals and fluctuation noises.

Introduction. LChM signals, which can be used to make simultaneous measurements of distance and velocity with a high resolution [1, 2] are often used in radar and radio navigation systems. We will study signals with symmetrical linear frequency modulation [3]. The reception is usually made with correlation filtering treatment [4]. In this the distance to the target corresponds to the telemetric increase in the frequency for the transformed signal [1].

In many problems when there are many targets, the parameters must be determined for that target whose distance is minimum (maximum). When signals with symmetrical LChM are used the problem is reduced to isolating that component from the spectrum for the telemetric frequencies which has the minimum (maximum) frequency and measuring its parameters. The necessity of isolating the outside frequencies in the signal's spectrum arises in other problems also. The solution for such a nonstationary, nonlinear problem, is associated with significant difficulties because of the pulsed nature of the input action and the lack of a priori information

about the parameters for the useful and interfering signals.

This article is devoted to a study of the operation of a dynamic discriminator for the outside frequencies (VKCh) of a signal's spectrum in the presence of fluctuation noises. Analytical expressions are found, with some simplification, for determining the behavior of VKCh for low perturbations. The main part of the studies was carried out using a mock-up of the high frequency part of the VKCh on the "elektron" analog computer.

Functional scheme for the unit. When interfering signals and fluctuation noises are present at the input to the unit the synthesis for the optimum discriminator of the wanted signal under conditions of an absence of a priori information about the parameters for the interfering signals results in rather complicated structural schemes. A version of the quasioptimum unit, analogous to that studied in [1] was achieved as the result of certain simplifications. The functional scheme for such a VKCh is shown in Fig. 1.

1) Input
2) Output
3) 1st channel
4) 2nd channel

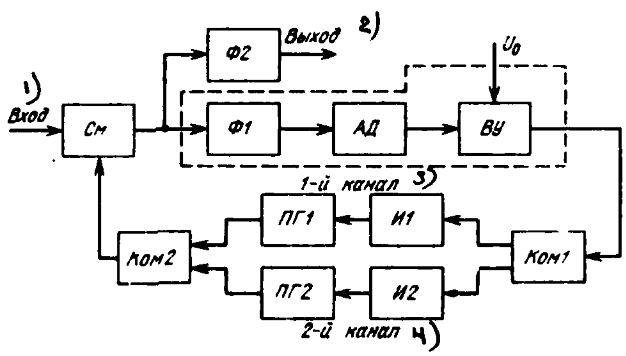

Fig. 1

In the usual case the unit contains two identical channels which are switched on alternately in the closed control

system by means of two commutators (Kom) in different half-periods of the modulation. All of the following discussions will refer to the pulsed operation of the VKCh with one channel. The unit's operation is illustrated in Fig. 2.

1) spectrum for the useful signal, 2) spectrum for the interfering signal, 3) spectrum of the noise

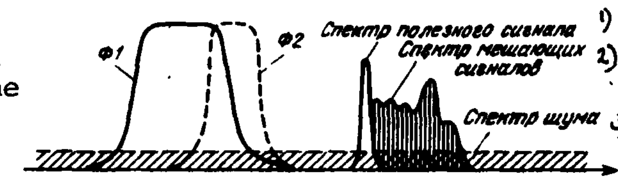

Fig. 2

At the beginning of the search for the wanted signal the spectra of the useful and interfering signals are outside of the transmission band of the 1st filter (Ф1). The spectrum of the useful signal should be closest to the tuning frequency of the filter. The voltage at the output to the amplitude detector (AD) in this case is proportional to the level of the fluctuation noises which pass through Ф1. The difference in the supporting voltage ($U_{оп}$) and the voltage at the output of the AD after the deducting device (VU) enters the integrator's (I) input. The voltage at the AD's output which is due to the action of the fluctuation noises should be less than $U_{оп}$.

There is a retuning of the frequency in the reorganized heterodyne (PG) due to the action of the voltage from the output of the integrator which changes linearly so that the spectrum for the signal, which is converted in the mixer (Sm), shifts along the frequency scale in the direction of the tuning frequency for Ф1. This shift stops when the spectrum for the converted signal coincides with the transmission band for Ф1 to such a degree that the voltage from the AD's output is comparable with the supporting voltage. In this

state the outside frequencies of the spectrum for the converted signal should not be outside the limits of the transmission band for filter $\Phi 2$.

Study of the structural scheme for the unit. The main difficulty in building the structural scheme for the unit is related with finding an equivalent transmission function along the envelope for $\Phi 1$. In order to ensure the necessary suppression of the interfering signals the slope for the amplitude-frequency characteristic (AChKh) should be quite steep. This is accomplished by using multicell, high-quality filters, and finding an equivalent transmission function for the multicell filter for the radio signal, whose frequency coincides with the slope for the AChKh filter leads to the expressions which are described by high order differential equations. In this study an equivalent filter circuit was used with respect to an envelope in the form of series-connected delay units and an aperiodic, first order cell. The validity of such a simplified representation is confirmed by an analysis of the intermediate characteristic of a filter which is obtained upon switching on a signal whose frequency coincides with the slope for the AChKh filter.

A preliminary analysis of the pulsed conditions for the operation of the VKCh shows that as the stability of the unit is increased the duration and the moment for switching of the channels by the 1st commutator (Kom 1) should differ somewhat from the intervals for the operation of the 2nd commutator (Kom 2) (Fig. 3). This is explained by the presence of pure delay in the structure of the discriminator (Υ_i) and the aperiodic cell. Taking the foregoing into consideration, the structural scheme for the given unit (single channel) can be represented in the form of the scheme (Fig. 4) on which Y(t) is the input action, X(t) is the signal for the

error, Z(t) is the output signal, $K_1(X)$ is the transmission coefficient for the discriminator which depends on the signal for the error, τ_1 and T_1 are the delay and the time constant for the equivalent cells in the discriminator, K_1 is the transmission coefficient for the integrator, and K_2 is the steepness for the characteristic of the reorganized heterodyne. Since such a system for the automatic control is nonlinear, because of the nonlinearity of the discriminator's characteristics, its behavior is studied for the assumption that the signal for the error is small. In this case the characteristic for the discriminator can be considered to be linear. An analysis of the linearized system (Fig. 4) was made using the approximate equations and the D-transformation method [5]. The results of the study are valid for the condition that the input action changes more slowly than the duration of the interval γ_1 (Fig. 3.).

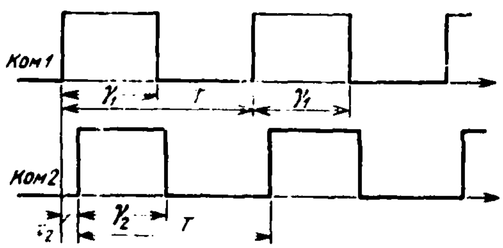

Fig. 3

In converting to dimensionless characteristics the transmission coefficients for the continuous parts of the given system are written in the following form:

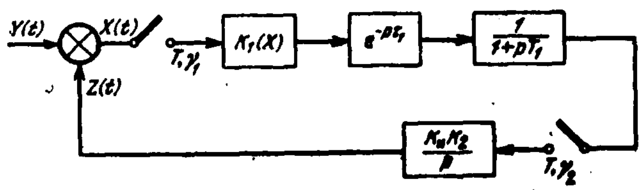

Fig. 4

$$\tilde{K}_{н1}(q)=\frac{K_1}{q+\beta}e^{-q\bar{\tau}_1}, \qquad (1)$$

$$\tilde{K}_{н2}(q)=\frac{K_2\beta_н}{q}. \qquad (2)$$

Here $\tilde{K}_{н1}(q)$, $\tilde{K}_{н2}(q)$ are the transmission coefficient for the given continuous parts of the system which are included, respectively, between the 1st and 2nd commutators and the output: $q=pT$; $\beta=T/T_1$; $\bar{\tau}_1=\tau_1/T$; $\beta_н=T/T_н$; $T_н$ are the integration times. By making the necessary transformations we find the transmission function for an open pulsed system

$$K^*(q,\varepsilon)=K_0\left[1-\frac{e^q-e^{\beta(1-\gamma_1)}}{e^q-e^{-\beta}}e^{-\beta(\bar{\tau}_2-\bar{\tau}_1)}\right]\times$$
$$\times\left(\varepsilon-\bar{\tau}_2+\frac{\gamma_2}{e^q-1}\right), \qquad (3)$$

where $\bar{\tau}_2=\tau_2/T$ (see Fig. 3) $K_0=K_1K_2\beta_н$.

The transmission function for the closed pulsed system is equal to

$$K_3^*(q,\varepsilon)=$$
$$=\frac{K_0\left[1-\frac{e^q-e^{\beta(1-\gamma_1)}}{e^q-e^{-\beta}}e^{-\beta(\bar{\tau}_2-\bar{\tau}_1)}\right]\left(\varepsilon-\bar{\tau}_2+\frac{\gamma_2}{e^q-1}\right)}{1+K_0\left[1-\frac{e^q-e^{\beta(1-\gamma_1)}}{e^q-e^{-\beta}}e^{-\beta(\bar{\tau}_2-\bar{\tau}_1)}\right]\left(\frac{\gamma_2}{e^q-1}-\bar{\tau}_2\right)}. \qquad (4)$$

The equation for the image for the output value has the form

$$Z^*(q,\varepsilon)=K_3^*(q,\varepsilon)F^*(q,0), \qquad (5)$$

where F*(q, 0) is the image of the given external action.

The Rauss-Gurvits algebraic criterion was used to study the stability of the system. It can be used to judge the stability of the system from the coefficients for the characteristic polynomial [5]. The use of this criterion gives the following system of inequalities which determine the limits of stability:

$$\bar{\tau}_2 > \bar{\tau}_1 + \frac{1}{\beta} \ln\left(\frac{e^{\beta} - e^{\beta\gamma_1}}{e^{\beta} - 1}\right), \tag{6}$$

$$K_{0\,\text{гр}\,1} < \left[\left(1 - \frac{e^{\beta} + e^{\beta\gamma_1}}{e^{\beta} + 1}\right) e^{\beta(\bar{\tau}_2 - \bar{\tau}_1)} \times \right.$$

$$\left. \times \left(\bar{\tau}_2 + \frac{\gamma_2}{2}\right)\right]^{-1}, \tag{7}$$

$$K_{0\,\text{гр}\,2} < \left[\bar{\tau}_2\left(1 + \frac{e^{\beta(\gamma_1 + \bar{\tau}_1 - \bar{\tau}_2)} - e^{\beta(1 + \bar{\tau}_1 - \bar{\tau}_2)}}{e^{\beta} - 1}\right) + \right.$$

$$\left. + \gamma_2\left(\frac{e^{\beta(\gamma_1 + \bar{\tau}_1 - \bar{\tau}_2)} - 1}{e^{\beta} - 1}\right)\right]^{-1}. \tag{8}$$

The inequality (6) determines the dependence of the time the 2nd commutator is switched on on the delay τ'_1, the duration γ_1 and th time constant for the aperiodic cell T_1. The relationship $\bar{\tau}_2 = \bar{\tau}_1 + 1/\beta$ which always satisfies the inequality (6), was used to determine the stability limits according to the inequalities (7) and 8.

The dependence of the limiting amplification coefficients on β are shown in Fig. 5 for different values of $\tilde{\tau}_1$. The region of stability lies below the corresponding limits. In Fig. 5 it follows that as β increases, the reserve for the stability at first increases and then decreases, in which case with an increase in β, beginning with some value, it does not change. As the delay time increases the stability of the system increases.

<u>The results of modeling</u>. Since the modeling of the VKCh units in terms of the envelope is approximate and related, in the given case, to significant technical difficulties, the operation of the unit was studied using a high frequency part of the VKCh mock up and an analog computer. The slope of the AChKh for the 5-resonator electromechanical filter of the EMFDP-500 - 0.5 s type was used to form the discriminating

characteristic in the unit. It has a transmission band at the 0.7 level equal to 500 Hz and a mean tuning frequency of f_0 = 500 kHz.

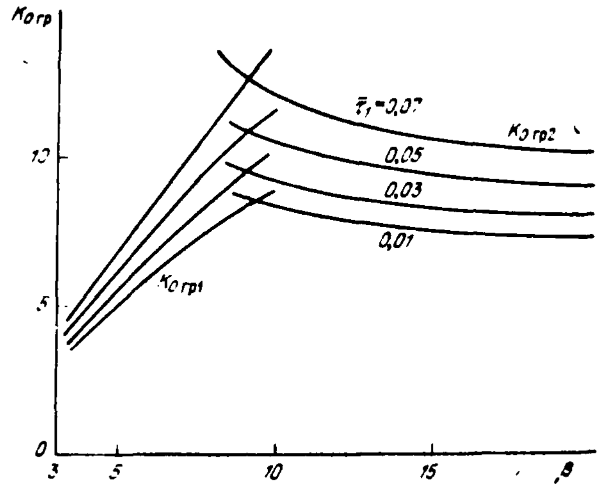

Fig. 5

A study was made of the intermediate and established operating conditoins for the VKCh for the input action of a jump. The system was studied for the action of the useful signal alone, the sum of the useful and interfering signals, and the sum of the useful and interfering signals and the fluctuation noises.

The studies that were made on the stability showed that the results of the analytical study coincide with the result obtained from modeling. This confirmed the validity of the simplified, equivalent structural scheme used above for the discriminator. A study was made of the intermediate

and established operating conditions for the VKCh for the input action which changes according to a linear symmetrical cross cut rule.

The results of the study are shown by the curves in Fig. 6 and they correspond to the following conditions:

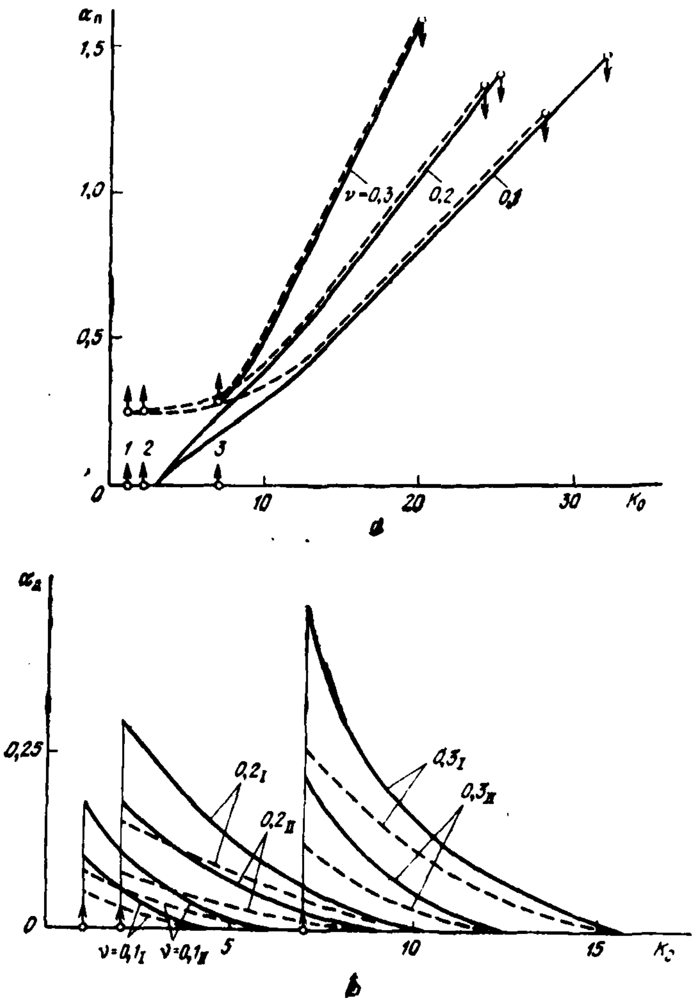

Fig. 6

- the mutual position of the carrying frequencies for the useful and interfering signals are given in Fig. 7.

- the ratio of the amplitudes of the acting signals is:

$$A_0 / A_1 = 1/3; \; A_0 / A_2 = 1/2;$$

- the ratio of the powers for the wanted signal and the fluctuation noise in the transmission band of the first filter, is equal to 500 Hz, is taken as equal to 5. This value was determined from the permissible level for the fluctuation noises at the input of the device for the subsequent treatment of the signals.

- the working point for the discriminator corresponds to the level of 0.5 on the AChKh's slope for the 1st filter.

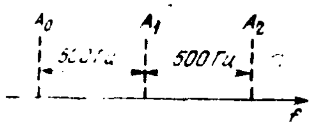

Fig. 7

The dependence of the oscillation amplitude of the error's signal on the amplification coefficient for the system is shown in Fig. 6a for three different rates ν for the "amicable" change in the frequency of the input signals. The dependence which corresponds to the action of a single useful signal and the sum of the useful and interfering signals at the input to the system is shown by the solid line. These curves basically coincide with one another which confirms the efficiency of the VKCh's operation. The dependence which corresponds to the overall action of the useful signal, the interfering signals, and the fluctuation noises is shown by the dotted line.

The dependencies for the dynamic tracking error for the system on the amplification coefficient are shown in Fig. 6b for three different rates of "amicable" change in the frequency of the output signals. In this case the following designations were used:

$$\alpha_п = \Delta f_п / \Pi_р; \quad \alpha_д = \Delta f_д / \Pi_р; \quad \nu = \sqrt{\mu} / \Pi_р,$$

where Δf_n is the range for the periodic oscillations for the error's signal (Hz), Δf_η is the dynamic error which is established for the system (Hz); Π_p = 250 Hz is the frequency band for the "working" part of the discriminator's characteristic (determined from the level from 0.1 to 0.9 for the AChKh's slope for the first filter); μ is the rate of change of the input action [Hz/sec]. The sign ♂ in Fig. 6 corresponds to the minimum amplification coefficient for which capture conditions set in with the subsequent tracking of the changing input action. The sign ♀ determines the maximum amplification coefficient for the system for which tracking is interrupted.

The index I corresponds to the tracking conditions for the half wave which is "going out of" the transmission band of the first filter for the linear change in the input action and the index II corresponds to the "incoming" half wave. As a conclusion to the article we can draw the following conclusions:

1. The system that was studied has a high stability both in terms of the interfering signals and the fluctuation noises.

2. The behavior of the system for the action of a single, useful signal differs very little from its behavior of the action of the sum of the useful and interfering signals, which exceeds the useful signal by more than a factor of ten.

3. The behavior of the system, for the action of the sum of the useful signal, interfering signals, and fluctuation noises at the input of the system, differs from the case in

which only the useful signal acts on the input only for small values of the amplification coefficient, provided we do not consider the slight shift in the limits for the interruption of tracking (Fig. 6a).

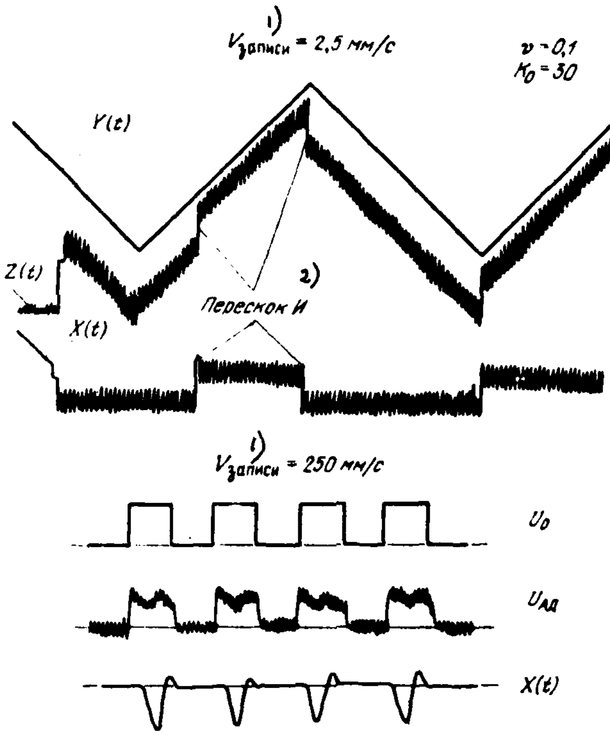

Fig. 8

1) record, 2) jump

4. When interfering signals are present at the input to the system and the amplification coefficient for the system is close to the limit for interruption of tracking, the phenomenon of "jumps" may occur in the tracking because of the high level for the interfering signal (Fig. 8).

5. The size of the dynamic error in tracking depends on the sign of the change for the input action.

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Possibilities for Improving the Interference Freedom for FM Systems Using Filters with Variable Parameters

O. E. Abramyants

The freedom from interference was studied for radar FM systems with SSPF and an evaluation of the gain in the signal/noise ratio is given, obtained from the use of such filters.

We know that complex FM signals which are widely used in radar devices are adequate filters with variable parameters [1-3, 6]. Because of the high phase sensitivity they can be used to improve the accuracy for measuring the signal's parameters for the action of various interferences. One such filter, which transmits complex FM signals without distorting the shape, is the structural-signal parametric filter (SSPF) [1, 2].

We know that the interference spectrum and the signal/noise ratio at the output to a standard filter depends only on the transmission band and does not depend on the FM signal. At the output to the SSPF (or any other parametric filter) the interference spectrum and the signal/noise ratio depends greatly on the FM signal since this law is uniquely related to the law for the change in the filter's parameters.

Let us assume that an additive mixture of a reflected signal $S_{oTp}(t)$ and interference $n(t)$ arrives at the input to the SSPF

$$x(t) = s_{\text{отр}}(t) + n(t),$$

then, at the output to the SSPF we will have

$$x_{вых}(t) = \int_{-\infty}^{t} x(\xi)\, g(t, \xi)\, d\xi = \int_{-\infty}^{t} s_{отр}(\xi)\, g(t, \xi)\, d\xi +$$

$$+ \int_{-\infty}^{t} n(\xi)\, g(t, \xi)\, d\xi = s_{вых}(t) + n_{вых}(t),$$

$g(t, \xi)$ is the pulsed, transition characteristic of the SSPF [2]; $s_{вых}(t)$ is the signal to the filter's output in the absence of the interference, and $n_{вых}(t)$ is the interference at the filter's output.

For a spectral representation the signal at the output to the filter will have the form

$$s_{вых}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_c(\omega)\, K(\omega)\, e^{j\omega\tau(t)}\, d\omega, \qquad (1)$$

where $A_c(\omega)$ is the spectrum for the reflected signal on the frequency scale corrected for the filter being used [2, 3], $K(\omega)$ is the corrected transmission function of the filter being used, and $\gamma(t)$ is a function which determines the law for the phase change for the FM signal.

The spectrum for the reflected signal on the frequency scale which has been corrected for the receiver SSPF can be found by expanding the reflected signal in terms of a system of functions of the intrinsic filter base [5]. If the probe signal is given in the form

$$s(t) = U_c\, e^{j\omega_0\tau(t)},$$

then the corrected spectrum for the reflected signal, which is held for a time $t_з$ which has a Doppler shift for the frequency Ω equal to

$$A_c(\omega) = U_c \int_{-\infty}^{\infty} e^{j\omega_0\tau(t-t_з)}\, e^{j\Omega(t-t_з)}\, e^{-j\omega\tau(t)}\, d\tau, \qquad (2)$$

in which U_c is the voltage amplitude for the reflected signal.

In the absence of a phase shift for the reflected signal, i.e., for $t_з = 0$ and $\Omega = 0$, it follows from (2) that

$$A_c(\omega) = 2\pi U_c \delta(\omega_0 - \omega).$$

By substituting this expression in equation (1) and assuming that $K(\omega) = 1$ in the corrected frequency band $\omega_0 \pm \Delta\omega_0$ we get

$$s_{вых}(t) = U_c e^{j\omega_0(t)},$$

i.e., in the absence of a phase shift the filter which is used does not distort the signal's shape. The normalized value of the modulus for the output voltage, in this case, is equal to one.

Improving the signal/noise ratio. Of all of the forms of interference in active radio counteraction the masking, trapping, selection, and mutual interference must be considered the most serious. The signal/noise ratio at the output of the SSPF for the action of a masking interference n(t) on the input in the form of normal, white noise was determined in [2 and 3]. The gain in the signal/noise ratio at the output of a narrow band parametric filter in this case is equal to the index of the FM parameters for the filter in the receiver [3].

$$\eta_\Phi = \frac{(P_c / P_ш)_{вых}}{(P_c / P_ш)_{вх}} = \beta. \qquad (3)$$

Let us determine the signal/noise ratio at the output of the SSPF for the action of selective (or mutual), wideband

FM interference [jamming] at the input which is analogous to the probing signal but which does not coincide with it in phase. Let us assume, for definitiveness, that the signal's interference's frequency are modulated according to a harmonic law and that the initial value of the frequencies coincide:

$$\tau_c(t)=t+t_{01}\sin\Omega_{01}t;\ \tau_n(t)=t+t_{02}\sin\Omega_{02}t;$$
$$s_{\text{отр}}(t)=U_c\,e^{j\omega_0\tau_c(t)};\quad n(t)=U_{\text{чм}}\,e^{j\omega_0\tau_n(t)},$$

where Ω_{01} and Ω_{02} are the frequencies for the modulation of the signal and of the interference, respectively and ω_0 is the initial frequency of the signal and of the interference.

The spectrum for the FM interference on the frequency scale corrected for the receiver's SSPF is equal to

$$A_{\text{чм}}(\omega)=\int_{-\infty}^{\infty}n(t)\,e^{-j\omega\tau_c(t)}d\tau=$$
$$=U_{\text{чм}}\int_{-\infty}^{\infty}\dot{\tau}_c(t)\,e^{j\omega_0\tau_n(t)}\,e^{-j\omega\tau_c(t)}dt=$$
$$=2\pi U_{\text{чм}}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}J_n(\omega_0t_{02})J_m(\omega_0t_{01})\times$$
$$\times\left(1+\frac{m\Omega_{01}}{\omega}\right)\delta(\omega_0-\omega+n\Omega_{02}-m\Omega_{01}).$$

Assuming (as in the first case) that $K(\omega)=1$ in the corrected frequency band $\omega_0\pm\Delta\omega_0$, at the output to the SSPF we get

$$u_{\text{чм вых}}(t)=\frac{1}{2\pi}\int_{\omega_0-\Delta\omega_0}^{\omega_0+\Delta\omega_0}A_{\text{чм}}(\omega)K(\omega)\,e^{j\omega\tau_c(t)}d\omega=$$
$$=U_{\text{чм}}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left(1+\frac{m\Omega_{01}}{\omega_0+n\Omega_{02}-m\Omega_{01}}\right)J_n(\omega_0t_{02})\times$$
$$\times J_m[(\omega_0+n\Omega_{02}-m\Omega_{01})t_{02}]\exp[j(\omega_0+n\Omega_{02}-$$
$$-m\Omega_{01})\tau_c(t)].\qquad(4)$$

In order to get the maximum signal/noise ratio a filter must be used which has a minimum corrected transmission band whose value is determined by the condition for the transmission of the expected Doppler shift Ω:

$$\Omega < \Delta\omega_0 < \Omega_{01}. \qquad (5)$$

The worst case is that in which the modulation frequency for the signal and interference are close to one another:

$$\Omega_{01} \simeq \Omega_{02}. \qquad (6)$$

Assuming that in the centimeter range (in which RLC [radar systems] usually operate) the following holds true

$$\omega_0 \gg \Omega_{01}, \qquad (7)$$

and also that equations (5) and (6) hold true, we get from equation (4)

$$u_{\text{чм вых}}(t) = U_{\text{чм}} J_0(\beta_1) J_0(\beta_2) e^{j\omega_0\tau_c(t)},$$

in which $\beta_1 = \omega_0 t_{01}$ and $\beta_2 = \omega_0 t_{02}$ – are the indices for the FM signal and the interference, respectively.

If the receiver's filter is phased with the signal then the gain in the signal/noise ratio for the action of a wideband FM interference is equal to

$$\gamma_{\text{чм}} = \frac{(P_c / P_{\text{чм}})_{\text{вых}}}{(P_c / P_{\text{чм}})_{\text{вх}}} = \frac{1}{J_0^2(\beta_1) J_0^2(\beta_2)}. \qquad (8)$$

For $\beta_1 \gg 1$ and $\beta_2 \gg 1$ equation (8) can be simplified by using the asymptotic expression for the Bessel function

$$\eta_{\text{нм}} \geqslant \pi^2 \beta_1 \beta_2 / 4. \qquad (9)$$

The expression (9) shows that the gain in the signal/noise ratio is sufficiently large even under the worst conditions for reception when, in addition to the phase, the signal and interference coincide in all parameters. The coincidence for the current phase in unlikely.

Another type of selective (mutual) interference is an oscillation centered in the spectrum. If the interference has the form

$$n(t) = U_{c\,n1}\, e^{j\omega_{c\,n} t},$$

then its spectrum on the frequency scale corrected for the receiver's SSPF is equal to

$$A_{c\,n\,1}(\omega) = 2\pi U_{c\,n1} \sum_{n=-\infty}^{\infty} \frac{\omega + n\Omega_0}{\omega} J_n(\omega t_0)\, \delta(\omega_{c\,n} - \omega + n\Omega_0).$$

If it is assumed that we have the same conditions (5) and (7) then at the output for the SSPF we get

$$u_{c\,n1\,\text{вых}}(t) = U_{c\,n1}\, J_0(\omega_{c\,n} t_0)\, e^{j\omega_{c\,n} \tau_c(t)}, \qquad (10)$$

where $\tau_c(t)$ is the law for the change in the current phase for the filter's resonance frequency tuned to the reflected signal.

Equation (10) is valid only in the case in which the interference's frequency is in the corrected transmission band for the receiver's SSPF, i.e., when $\omega_{c\,n} < \omega_0 \pm \Delta\omega_\phi$.

Taking (7) into consideration, we can write

$$\omega_{c\,n} t_0 \simeq \omega_0 t_0 = \beta_1.$$

If the parameters of the filter are modulated according to a harmonic law, then the interference's power at the filter's output is

$$P_{c\,n1} = [U_{c\,n1} J_0(\beta_1)]^2 \leqslant U^2_{c\,n1} \frac{2}{\pi \beta_1}. \qquad (11)$$

Comparing equations (8) and (11) shows that we can find the signal/noise ratio for the action of an interference centered in the spectrum from (8) by assuming $\beta_2 = 0$. The gain in the signal/noise ratio for the action of an interference centered in the spectrum and for a harmonic law for the change in the filter's parameters is equal to

$$\eta_{c\,n1} = \frac{(P_c / P_{c\,n1})_{\text{вых}}}{(P_c / P_{c\,n1})_{\text{вх}}} \geqslant \frac{\pi}{2} \beta_1. \qquad (12)$$

Let us consider a different case in which the interference which is centered in the spectrum acts on the input to the SSPF whose resonance frequency changes according to the linear rule $\tau_c(t) = t + \mu t^2/\omega_0$ and whose initial frequencies coincide. The corrected spectrum for the interference is equal to

$$A_{c\,n2}(\omega) = U_{c\,n2} \int\limits_{-T/2}^{T/2} e^{j\omega_0 t} e^{-j\left(\omega t + \frac{\omega}{\omega_0}\mu t^2\right)} d\tau =$$
$$= U_{c\,n2} \sqrt{\pi/2\mu} \{[C(v_2) - C(v_1)] - j[S(U_2) - S(v_1)]\},$$

where T is the period for the frequency modulation for the filter, $\mu = \Delta\omega/2T$ — is the rate of change for the filter's frequency, C(v) and S(v) are Fresnel integrals,

$$v_2 = -v_1 = \sqrt{\Delta f T/2}; \quad \Delta f = \Delta\omega / 2\pi.$$

By using the asynptotic expression for the Fresnel integral for v $\gg$ 1 and taking into account the relationship $C(-v) = -C(v)$, $S(-v) = -S(v)$ we get the following expression for the interference at the output to an SSPF with linear FM

$$u_{c\,n2\,\text{вых}}(t) = U_{c\,n2}\frac{1}{\Delta f T}\,\frac{\sin\Delta\omega_0\tau_c(t)}{\Delta\omega_0\tau_c(t)}\,e^{j\omega_0\tau_c(t)}.$$

Under the worst conditions

$$\frac{\sin\Delta\omega_0\tau_c(t)}{\Delta\omega_0\tau_c(t)} \to 1 \quad \text{for}\; t \to 0,$$

then the gain in the signal/noise ratio at the output to the SSPF is

$$\eta_{c\,n2} = \frac{(P_c/P_{c\,n2})_{\text{вых}}}{(P_c/P_{c\,n2})_{\text{вх}}} \geqslant (\Delta f\,T)^2, \qquad (13)$$

i.e., even for signals with a low base the gain in the signal/noise ratio at the output for the SSPF is significant.

<u>Measuring accuracy</u>. Let us evaluate the accuracy for measuring distance for an SSPF location system. The dispersion in the potential error in measuring distance for the action of a trapping interference in the form of normal white noise [4] is

$$\sigma_0^2 = \frac{1}{\Delta W^2 q},$$

where q is the signal/noise ratio at the filter's output and ΔW is the equivalent band which characterizes the frequency spectrum for the signal [4].

If we assume that the signal/noise ratio at the input to the receiving filter is equal to unity, then we can use

the value of the gain in the signal/noise ratio, found at the SSPF's output, directly in the expression for the dispersion. As was shown earlier, the signal/noise ratio at the SSPF's output increases β times for the action of a trapping interference in the form of normal white noise and therefore the dispersion in the potential error in measuring the distance, according to [3], is equal to

$$\sigma_0^2=\frac{1}{\Delta W^2\beta}=\frac{1}{\Delta W^2\Delta f T},$$

since the FM index can be represented as the base for the continuous signal $\beta=\Delta\omega/\Omega_0=\Delta fT$.

Thus, location systems with SSPF have the same potential accuracy in measuring the distance for a continuous signal as the pulsed system with an optimum (in Norse's terms) filter. The dispersion in the potential error for measuring distance for a nontracking FM range finder with compensation of the phase shift, according to [6] is

$$\sigma_1^2=\frac{3}{\Delta W^2 q}\left(1+\frac{1}{q}\right).$$

A comparison of this expression with an analogous expression for the SSPF can be used to derive the equation $\sigma_1^2=3\sigma_0^2(1+1/q)$.

The poor accuracy for the nontracking range finders is due to the nonoptimum, predetector treatment of the signal and the effect of the dispersion in the error of measuring the frequency from which the distance is measured in such systems.

The dispersion in the error for measuring the distance for the action of selective (mutual) interferences are equal to:

- for a harmonic law for the FM interference and the filter's parameters according to (9) $\sigma_{чм} = 2/\Delta W^2 \pi^2 \beta_1 \beta_2$;

- for an interference which is centered in the spectrum and a harmonic rule for the change in the filter's parameters according to (12) $\sigma^2_{c\,n1} = 1/\Delta W^2 (\Delta f T)^2$;

- for an interference which is centered in the spectrum and a linear rule for the change in the filter's parameters, according to (13) $\sigma^2_{c\,n2} = 1/\Delta W^2 (\Delta f T)^2$.

The total dispersion of the error in measuring distance can be achieved by adding the dispersion in the potential error to any of the dispersions in the error from the action of selective (mutual) interferences.

<u>Conclusion</u>. An analysis of the expressions which were found shows that the error in measuring the distance under the operating conditions for a system of active counteraction can be greatly reduced by using structural-signal parametric filters for treatment in continuous, radar FM systems. The use of such filters lets us approach the optimum treatment of the continuous FM signals with a broad base. It is very difficult to build the optimum (in Norse's terms) filter for the treatment of a continuous signal in the course of several thousands of modulation periods whereas it is not difficult to realize an SSPF for the treatment of a wide band, continuous FM signal.

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Analysis of the Effect of Quasidetermining Interferences on the Operation of a Receiver with a Coordinated LFM Filter

Yu. N. Babanov, Yu. P. Lebedev, and I. Ya Orlov

One of the problems in the electromagnetic compatibility for radioelectronic devices is to protect the receiver of LFM [linear frequency modulated] signals from the action of radio pulse interferences. We will make an analysis of the passage of an interference, in the form of a radio pulse with a Gaussian envelope.

$$n(t) = A_1 \exp(-t^2/2T_1^2) \exp j\omega_1 t, \qquad (1)$$

through a filter which is coordinated with the LFM signal

$$s(t) = A_0 \exp(-t^2/2T_0^2) \exp j(\omega_0 t + \mu t^2/2). \qquad (2)$$

The response for a linear filter which is coordinated with signal (2) to the action of an interference (1) is found from the superposition integral and it has the form

$$n_{\text{вых}}(t) = \exp[-(t+t_з)/2T_в^2] \exp j\{\omega_в t - \\ - \mu_в (t+t_з)^2/2 + \varphi_в\}. \qquad (3)$$

Here the output parameters $A_в$, $T_в$, $t_з$, $\omega_в$, $\mu_в$ $\varphi_в$ are independent of the time and the response of the LFM filter to the interference ... a high frequency oscillation with a linear frequency ...tion and a Gaussian envelope.

...ration of the interference at the filter's output ... where

$$K_T^2 = \frac{[(1+T_0'^2)\,T_1'^2]^2 + T_0'^6}{T_1'^2(1+T_0'^2)\,[(1+T_0'^2)\,T_1'^2 + T_0'^2]}, \qquad (4)$$

$$T_1' = \Delta\omega\, T_1; \quad T_0' = \Delta\omega\, T_0,$$

where $\Delta\omega = 1/\tau$ — is the deviation in the frequency for the LFM signal.

T_B does not depend on the detuning of the mean frequencies Δ and its magnitude at the output is always greater than at the input (i.e., $T_B \geqslant T_1$) and, moreover, the asynptotic relationships $\lim\limits_{T \to \infty} T_в = T_1$ и $\lim\limits_{T_1 \to 0} T_в = T_0$.

For each LFM for the coordinated filter there is an interference (1) of duration $T_1^* = \sqrt{\Delta\omega\, T_0' - 1}\,/\,\Delta\omega$ (Fig. 1), which gives the minimum response time $T_в^* = \sqrt{2/\mu} = T_0\sqrt{2}\,/\,\sqrt{\Delta\omega\, T_0}$, in which the values of T_B^* and T_1^* are only determined by the parameters of the filter T_0 and $\Delta\omega$.

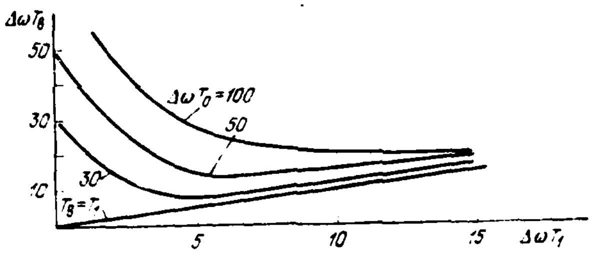

Fig. 1

The coefficient of linear frequency modulation for the interference at the output is

$$\mu_в = \frac{T_0'^4(1+T_0'^2)}{[(1+T_0'^2)\,T_1'^2 + T_0'^2]^2 + T_0'^6}\,\mu = K_\mu\,\mu. \qquad (5)$$

From which we can see that the LFM coefficient for the interference at the output assumes values which have an upper and lower limit ($\mu<\mu_{н}<0$) and which are not dependent on the detuning of the carriers for the signal's and interference's frequencies Δ.

Three segments can be observed on the curves shown in Fig. 2 (I, II, III). The functions $K_{\mu}=f(\Delta\omega T_1)$ can be used on the sections with either small (I and III) or large (II) increments. For example, for a duration $T_1 \simeq \tau$ (section I) an LFM Gaussian signal can be formed with a stable coefficient $\mu_{н} \simeq \mu$. For a duration of the input signal of $2\tau < T_1 < 10\tau$ (section II) an LFM Gaussian signal can be formed with a variable μ, in which case the base for the interference at the output is limited from both sides: $0<\mu_{н} T_{н}^2<\mu T_0^2$.

One of the essential phenomena which accompany the passage of a radio pulse through the LFM filter is the delay of the interference at the filter's output

$$t_з=\frac{T_0'^2 T_1'^2}{(1+T_0'^2)T_1'^2+T_0'^2}\,\frac{\Delta}{\mu}=K_0\frac{\Delta}{\mu}, \qquad (6)$$

in which the coefficient K_0 rapidly approaches unity as T_1' increases and $K_0 \simeq 1$ при $T_1 \geqslant 10\tau$. Thus, the appearance of the delay is due, first of all, to the presence of dispersion in the LFM in the filter and frequency detuning of the signal and the interference. Let us consider two specific cases.

1. The filter is coordinated with a signal which has a large base, i.e., $\Delta\omega T_0 \gg 1$; the duration of the interference is much greater than the correlation time of the signal, i.e., in the given designations $\Delta\omega T_1 \gg 1$. For an arbitrary detuning $\Delta=\omega_1-\omega_0$ the average frequencies can be written

$$n_{\text{вых}}(t) = \frac{A_1 T_1}{\sqrt[4]{T_1'^4 + T_0'^2}} \exp\left(-\frac{\Delta^2}{2\Delta\omega^2}\right) \times$$

$$\times \exp\left[-\frac{(t+\Delta/\mu)^2}{2(1+T_0'^2/T_1'^4)\,T_1^2}\right] \times \tag{7}$$

$$\times \exp j\left[\omega_1 t - \frac{T_0'^2}{T_1'^4 + T_0'^2}\,\frac{\mu}{2}\left(t+\frac{\Delta}{\mu}\right)^2 + \frac{\Delta^2}{2\mu} - \right.$$

$$\left. - \frac{1}{2}\operatorname{arc\,tg}\frac{T_1'^2}{T_0'}\right].$$

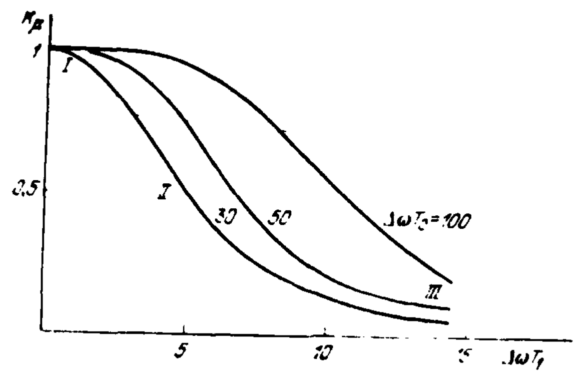

Fig. 2

It follows from (7) that the conditions $\Delta\omega T_0 \gg 1$ and $T_1 \gg \tau$ are not asymptotic for the output parameters of the interference $A_\text{в}$, $T_\text{в}$, $\mu_\text{в}$ и $\varphi_\text{в}$, since the ratio between the parameters T_0 and T_1 remains arbitrary. However, these conditions are sufficient for the mean frequency of the output signal to assume a maximum value equal to the mean frequency of the interference at the input $(\omega_\text{в} = \omega_1)$, and the delay time $t_з$ is determined only by the dispersion characteristic of the filter.

2. This specific case is completely asymptotic. Here, in addition to the condition $\Delta\omega T_0 \gg 1$ the duration of the interference is severely limited $T_1 \gg T_0$. Then

$$n_{\text{вых}}(t) = A_1 \exp\left(-\frac{\Delta^2}{2\Delta\omega^2}\right)\exp\left(-\frac{(t+\Delta/\mu)^2}{2T_1^2}\right)\times$$
$$\times \exp j\left\{\omega_1 t + \frac{\Delta^2}{2\mu} - \frac{\pi}{4}\right\}. \qquad (8)$$

In this case the response parameters for the interference at the filter's output are determined completely by the parameters of the filter or the corresponding parameters of the interference. In fact the amplitude $A_в = A_1 \mid K(j\omega)\mid$, the delay time $t_з$ and the phase shift $\Delta^2/2\mu$ are determined by the position of the working point for the dispersion characteristic of the filter and by the detuning. The duration of the interference with respect to the level is 0.6 and the mean frequency remains unchanged.

By comparing equations (7) and (8) and the conditions for which they were obtained, we find that the phase characteristic for the filter, and particularly its inertia, have no effect on the shape of the signal and on its spectral density for the condition $T_1 \gg T_0$.

Experiments made on the BESM-4 showed that the delay of the radio pulse at the output to the LFM filter can have quite a large value (Fig. 3)

The analysis which was made leads to the conclusion that it is possible to use this effect to combat radio pulse interference. The strobe time at the output to the LFM filter can be used in this case to filter the interferences which are tuned in frequency to the value

$$\Delta f > \Delta > \mu T_1 / K_0,$$

in which Δf is the tuning at which the filtration frequency

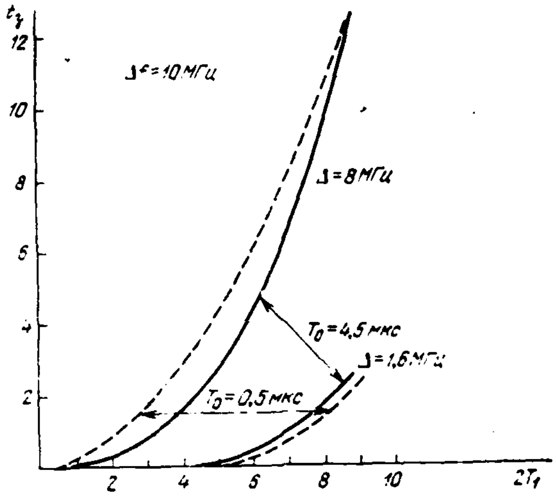

Fig. 3

is more effective than the intermittent strobing at the output of the LFM filter.

Effect of Amplitude Changes of a Signal in Detuned Linear Circuits of Communication Systems with Frequency and Phase Modulation

D.V. Ageyev and A. V. Zen'kovich

The passage of an FM signal with amplitude modulation was studied through linear circuits in front of the limiter. Using the most widely used circuit, i.e., a single oscillation circuit, the distortion was determined for the law of changes in frequency due to the effect of amplitude modulation during detuning by using the polynomial method under dynamic conditions. The nonlinearity of the phase characteristic and the unevenness of the frequency characteristic for the detuned circuit are taken into account. The dependence of the distortions which arise on the parameters of the FM signal and of the circuit was determined.

The study of the effect of amplitude changes of a signal in communications systems with frequency and phase modulation under dynamic conditions was first done in [1]. However, the rule for the changes in the frequency which occur because of AM distortions was only determined for the case of precise tuning of the selective circuits in the central frequency of the signal. Under real conditions this assumption may not be fulfilled. Taking the detuning into account under dynamic conditions is also important to evaluate the results of [2] in which the detuning, i.e., the lack of symmetry for the circuits characteristics with respect to the central frequency, is assumed to be the only reason for the effect of the amplitude modulation of the FM signal and the study was carried out under static conditions.

Let us consider the passage of an FM signal with AM of arbitrary shape

$$u_1 = A(t)\exp j[\omega_0 t + \varphi(t)] = U_1 \exp j\omega_0 t \qquad (1)$$

through a detuned, single oscillation circuit, assuming that the effective frequency band for the signal and, consequently, the variable detuning $\Delta\omega$ and the constant detuning $\Delta\omega_0$ are small as compared with the transmission band for the circuit, i.e., assuming that $\Delta\omega\tau < 1$ and $\Delta\omega_0\tau < 1$. In this case, by using the first terms of the corresponding Taylor series, we can write the modulus and the phase of the transmission coefficient of the circuit $\dot{K} = K\exp(-j\psi)$ approximately in the form

$$K = Q/\sqrt{1 + (\Delta\omega_0 + \Delta\omega)^2\tau^2} \simeq Q[1 - (1/2)\Delta\omega_0^2\tau^2 -$$
$$- \Delta\omega_0\Delta\omega\tau^2 - (1/2)\Delta\omega^2\tau^2],$$
$$\psi = \operatorname{arctg}(\Delta\omega_0 + \Delta\omega)\tau \simeq \Delta\omega_0\tau - (1/3)\Delta\omega_0^3\tau^3 +$$
$$+ \Delta\omega\tau - \Delta\omega_0^2\Delta\omega\tau^3 - \Delta\omega_0\Delta\omega^2\tau^3 - (1/3)\Delta\omega^3\tau^3.$$

By writing $K = K_1 K_2$, we will exclude the nondistorting quadrupole with a modulus of the transmission coefficient $K_1 = Q$ for the initial phase $\psi_{10} = \Delta\omega_0\tau - (1/3)\Delta\omega_0^3\tau^3$ and a constant delay $\tau_0 = \tau - \Delta\omega_0^2\tau^3$. from the discussion. The remaining circuit being studied has, as a first approximation, the following transmission coefficient

$$\dot{K} = 1 - (1/2)\Delta\omega_0^2\tau^2 - \Delta\omega_0\Delta\omega\tau^2 + j\Delta\omega_0\Delta\omega^2\tau^3 -$$
$$- (1/2)\Delta\omega^2\tau^2 + (1/3)j\Delta\omega^3\tau^3.$$

The polynomial [3, 4] is used to determine the complex envelope for the output voltage form the circuit U_2 for an input voltage (1). According to this

$$U_2 = (1 - (1/2)\Delta\omega_0^2\tau^2)U_1 + j\Delta\omega_0\tau^2 U_1^{(I)} -$$
$$- j\Delta\omega_0\tau^3 U_1^{(II)} + (1/2)\tau^2 U_1^{(II)} - (1/3)\tau^3 U_1^{(III)},$$

where

$$U_1 = A(t)\exp j\varphi(t);\quad U_1^{(I)} = \exp j\varphi(t)\,[A^{(I)}(t) - j\varphi^{(I)}(t)];$$
$$U_1^{(II)} = \exp j\varphi(t)\,\{A^{(II)}(t) + 2j\varphi^{(I)}(t) + j\varphi^{(II)}(t)A(t) -$$
$$- A(t)[\varphi^{(I)}(t)]^2\};$$
$$U_1^{(III)} = \exp j\varphi(t)\,\{A^{(III)}(t) + 3j\varphi^{(I)}(t)A^{(II)}(t) -$$
$$- 3A^{(I)}(t)[\varphi^{(I)}(t)]^2 - 3A(t)\varphi^{(I)}(t)\varphi^{(II)}(t) -$$
$$- jA(t)[\varphi^{(I)}(t)]^3 + jA(t)\varphi^{(III)}(t)\}.$$

For precise tuning $U_{20} = U_1 + (1/2)\tau^2 U_1^{(II)} - (1/3)\tau^3 U_1^{(III)}$
The results from studying this case are given in [1]. By using them we can write

$$U_{21} = U_2 - U_{20} = -(1/2)\Delta\omega_0^2\tau^2 U_1 +$$
$$+ j\Delta\omega_0\tau^2 U_1^{(I)} - j\Delta\omega_0\tau^3 U_1^{(II)}. \qquad (2)$$

This expression characterizes the distortions of the complex envelope which determine the detuning $\Delta\omega_0$. In this problem the distortions in the law for the changes in the frequency due to the effect of AM are of interest. For a sinusoidal AM and FM

$$A(t) = 1 + m\sin\Omega_1 t,\quad \varphi(t) = \beta\sin\Omega_2 t. \qquad (3)$$

By substituting U_1, $U_1^{(I)}$, and $U_1^{(II)}$ into (2), which were calculated taking the conditions in (3) into account, and by carrying out the transformation, we find

$$U_{21} = \exp j\beta \sin \Omega_2 t \{-(1/2)\Delta\omega_0^2\tau^2(1 + m\sin\Omega_1 t) -$$
$$-\Delta\omega_0\tau^2\beta\Omega_2\cos\Omega_2 t(1 + m\sin\Omega_1 t) +$$
$$+ 2\Delta\omega_0\tau^3\beta\Omega_2 m\Omega_1\cos\Omega_1 t\cos\Omega_2 t - \Delta\omega_0\tau^3\beta\Omega_2^2 \times$$
$$\times \sin\Omega_2 t(1 + m\sin\Omega_1 t) + j[\Delta\omega_0\tau^2 m\Omega_1\cos\Omega_1 t +$$
$$+ \Delta\omega_0\tau^3 m\Omega_1^2\sin\Omega_1 t + \Delta\omega_0\tau^3\beta^2\Omega_2^2\cos^2\Omega_2 t +$$
$$+ \Delta\omega_0\tau^3 m\beta^2 m\beta^2\Omega_2^2\cos^2\Omega_2 t\sin\Omega_1 t]\}. \qquad (4)$$

From this, for detuning to the distortions in the law for the changes in the frequency for the tuned circuit found in [1], which are characterized by a coefficient for nonlinear distortions γ_1 and a coefficient for combination distortions γ_2, in which for $\Omega_1 \simeq \Omega_2$

$$\gamma_1 = (1/4)\beta^2\Omega_2^3\tau^3, \quad \gamma_2 = (3/2)m\Omega_2^2\tau^2, \qquad (5)$$

the value for the change in the frequency $\Delta\omega(t)$ is added. In the first approximation we get, from (4)

$$\Delta\omega(t) = -\Delta\omega_0\tau^2 m\Omega_1^2\sin\Omega_1 t +$$
$$+ \Delta\omega_0\tau^3 m\Omega_1^3\cos\Omega_1 t - \Delta\omega_0\tau^3\beta^2\Omega_2^3\sin 2\Omega_2 t +$$
$$+ (1/4)\Delta\omega_0\tau^3 m\beta^2\Omega_2^2(2\Omega_2 + \Omega_1)\cos(2\Omega_2 + \Omega_1)t. \qquad (6)$$

Here the first two terms determine the distortions, summed with - in the quadrature, which only depend on the change in the amplitude of the input voltage (1). In order to explain the physical meaning of these components for the distortions let us consider the passage of a signal without FM

$$u_3 = (1 + m\cos\Omega_1 t)\sin\omega_0 t = \sin\omega_0 t +$$
$$+ (1/2)m\sin(\omega_0 + \Omega_1)t + (1/2)m\sin(\omega_0 - \Omega_1)t$$

through a linear circuit with an arbitrary transmission coefficient (Fig. 1) standardized with respect to the carrier frequency $\omega_0 (K_0 = 1, \varphi_0 = 0)$. At the output for such a circuit

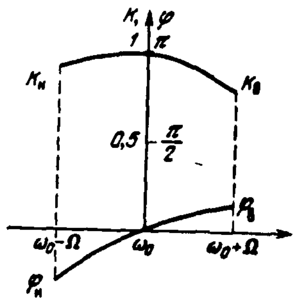

Fig. 1

$$u_4 = \sin \omega_0 t + (1/2) m K_в \sin [(\omega_0 + \Omega_1) t + \varphi_в] + \\ + (1/2) m K_н \sin [(\omega_0 - \Omega_1) t + \varphi_н].$$

By adding and subtracting $(1/2) m K_в \sin [(\omega_0 - \Omega_1) t - \varphi_в]$, we will write u_4 in the form of the sum of the AM signal having a frequency ω_0 and one component of the frequency $\omega_0 - \Omega_1$

$$u_4 = [1 + m K_в \cos (\Omega_1 t + \varphi_в)] \sin \omega_0 t + \\ + (1/2) m A_1 \sin [(\omega_0 - \Omega_1) t + \varphi_1],$$

where $A_1 = \sqrt{K_в^2 + K_н^2 - 2 K_в K_н \cos (\varphi_в + \varphi_н)}$; $\varphi_1 = \varphi_н + $

$$+ \operatorname{arc\,tg} \frac{K_в \sin \varphi_в}{K_н - K_в \cos \varphi_в}.$$

We get the following for the phase modulation of the resulting oscillation corrected to the frequency ω_0

$$\varphi_2 (t) = \operatorname{arctg} \frac{m A_1 \sin (\Omega_1 t - \varphi_1)}{2 [1 + m K_в \cos (\Omega_1 t + \varphi_в) + (1/2) m A_1 \cos (\Omega_1 t - \varphi_1)]},$$

and for the frequency modulation taking into account the smallness of m in the first approximation we get

$$\varphi_2^{(1)}(t) = (1/2)\, m\,\Omega_1\, A_1 \cos(\Omega_1 t - \varphi_1).$$

The deviation in the frequency for this voltage is

$$|\varphi_2^{(1)}(t)| = (1/2)\, m\,\Omega_1 \sqrt{K_{\text{в}}^2 + K_{\text{н}}^2 - 2K_{\text{в}} K_{\text{н}} \cos(\varphi_{\text{в}} + \varphi_{\text{н}})} =$$
$$= \sqrt{\Delta\omega_1^2 + \Delta\omega_2^2}.$$

Here the quantity $\Delta\omega_1$ is determined by the uneveness of the circuit's frequency characteristic and $\Delta\omega_2$ is determined by the nonlinearity in its phase characteristic, in which case

$$\Delta\omega_1 = (1/2)\, m\,\Omega_1 (K_{\text{в}} - K_{\text{н}}),$$
$$\Delta\omega_2 = m\,\Omega_1 \sqrt{K_{\text{в}} K_{\text{н}}} \sin(1/2)(\varphi_{\text{в}} + \varphi_{\text{н}}). \qquad (7)$$

For the standardized frequency characteristic of the detuned circuit we have

$$K_{\text{в}} - K_{\text{н}} = 1/\sqrt{1 + (\Omega_1 + \Delta\omega_0)^2\tau^2} -$$
$$- 1/\sqrt{1 + (\Omega_1 - \Delta\omega_0)^2\tau^2} \simeq 2\Delta\omega_0\,\Omega_1\,\tau^2,$$

from which, taking (7) into account $\Delta\omega_1 = (1/2)\, m\,\Omega_1\, 2\Delta\omega_0\,\Omega_1\,\tau^2 = \Delta\omega_0\,\tau^2 m\,\Omega_1^2$, which coincides with the deviation for the frequency of the first component in equation (6). This component is due to the transformation of the amplitude modulation into frequency (phase) modulation because of the nonsymmetry of the frequency characteristic of the detuned circuit. Calculation gives

$$\varphi_{\text{в}} + \varphi_{\text{н}} = [\operatorname{arctg}(\Omega_1 + \Delta\omega_0)\tau - \operatorname{arctg}\Delta\omega_0\tau] -$$
$$- [\operatorname{arctg}(\Omega_1 - \Delta\omega_0)\tau - \operatorname{arctg}\Delta\omega_0\tau] \simeq$$
$$\simeq (1/3)\,[(\Omega_1 + \Delta\omega_0)^3\tau^3 - (\Omega_1 - \Delta\omega_0)^3\tau^3] \simeq 2\Delta\omega_0\,\Omega_1^2\tau^3.$$

According to (7) for $K_B = K_H \simeq 1$

$$\Delta \omega_2 = m\, \Omega_1 \sin \Delta \omega_0\, \Omega_1^2 \tau^3 \simeq m\, \Delta \omega_0\, \Omega_1^3 \tau^3,$$

which coincides with the deviation in the frequency for the second component in equation (6). This component is due to the lack of symmetry for the phase characteristic of the detuned circuit.

The third component in (6) depends only on the change in the frequency of the input signal (1). It determines the nonlinear distortions in the second harmonic which arise during detuning of the circuit. For the tuned circuit

$$\gamma_3 = \Delta \omega_0\, \beta\, \Omega_2^2 \tau^3; \quad \delta = \gamma_3 / \gamma_1 = 4\, \Delta \omega_0 / \beta\, \Omega_2. \qquad (8)$$

If it is assumed that for $\delta \leq 0.3$ the nonlinear distortions that were indicated can be ignored then, if follows from (8), that the detuning in this case should not exceed 8% of the value of the deviation in frequency. For large modulation indices β fulfilling this condition does not cause difficulties. For $\beta < 1$, the role of the nonlinear distortions with respect to the third (γ_1) and the second (γ_3) harmonic changes. In order that the effect of γ_1 by negligible, i.e., for $\gamma_1/\gamma_3 \leqslant 0,3$, to be true, the relationship $\beta\Omega_2 \leqslant 1,3\Delta\omega_0$, must be fulfilled, i.e., the frequency deviation must exceed the detuning by no more than 30%.

The second component in (6) can usually be ignored as compared with the first, for which the ratio of its amplitude to the useful frequency deviation for $\Omega_1 = 2\Omega_2$ is $\Delta = 4\Delta\omega_0 m\Omega_2\tau^2/\beta$, and for large modulation indexes for which $\delta \leqslant 0,3$, $\Delta \leqslant 0,3$. $(m\Omega_2\tau)^2$. A comparison of the value for Δ with nonlinear dis-

tortions $\gamma_{21} = 3m\tau^2\Omega_2^2$ shows that for $\Omega_1 = 2\Omega_2$ $\Delta < 0{,}1\gamma_{21}$.
Therefore, for small detunings, for which the effect of the detuning on the nonlinear distortions can be ignored ($\delta \leqslant 0.3$), the first two components in equation (6) can be ignored.

Let us evaluate the relative effect of the first and third components in (6) for small modulation indices for which the detuning is great. $(\beta\Omega_2 \leqslant 1{,}3\Delta\omega_0)$.
If $\delta_1 = \gamma_3/\Delta$ 0,3, i.e., if

$$\beta \quad \frac{\Omega_1}{\Omega_2}\sqrt{\frac{0{,}33\,m}{\Omega_2\,\tau}},$$

then the main distortions are due to the effect of AM. However, if $\delta_1 \geqslant 3$ then the effect of AM, as compared with the additional nonlinear distortions can be compared.

By comparing the ratio of the amplitudes for the last component in (6) to the useful frequency deviation σ with the main distortions γ_1 for precise tuning of the circuit and for $\Omega_1 = 2\Omega_2$ we find that $\delta_2 = 6/\gamma_1 = \delta m$. For small detunings $\delta \leqslant 0{,}3$, δ_2 $0{,}3m$, and for large detunings the effect of the last component in (6) is also insignificant.

Thus, detuning mainly affects the increase in the nonlinear distortions for the law for the changes in frequency due to the parameters of the frequency modulation. The effect of AM appears to a lesser extent. Taking the lack of symmetry in the circuit's characteristics into account with respect to the central frequency of the signal, made under static conditions (in the absence of FM) [2] is completely insufficient for determining the effect of AM in the presence of FM.

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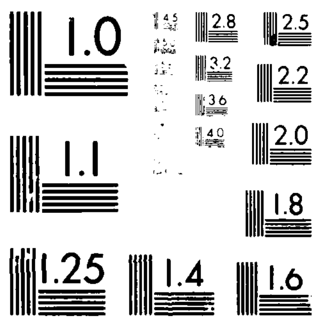

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Space Time Treatment of FM Signals

B.P. Burdzeyko and V.V. Shakhigil'dyan

The application of the theory of conditional Markov processes to the problem of synthesizing the optimum treatment of signals for multichannel viewing to the action of an interfering FM signal with specific statistical properties and an unknown position of the source of emission is discussed.

At the present time the problem of synthesizing the optimum treatment of signals for multichannel reception of useful and interfering signals occurs in different areas of technology. The known works in which the problem for the optimum space-time (space-frequency) filtration (detection) of the signals for quasidetermined or stationary Gaussian processes are [1-3].

In some cases it is necessary to solve the problem for the reception of nongaussian signals for which the model may be the Markov processes. The latter are widely used to describe FM signals.

In this work, the questions of synthesizing and analyzing the quality for the operation of receivers using the Markov model for the processses (particularly for the diffusion processes) will be studied. Particular attention is given to the problem of the reception of a signal of known shape with a nongaussian interfering signal with an unknown spatial position for the source. The methods for the statistics for Markov processes are used to find the effective solutions to various problems for an adequate model and for real processes.

1. Let us consider the structure of an optimum receiver of pictures for the action of the fields of the useful and interfering signals which are created by spatially separated sources, on the antenna system.

Let L be the discrete, ordered set which corresponds to the position in space of the elements of the antenna system, T is the observation time, and Y = Y(t) is the observation process on L and T

$$Y = (y_1, y_2, \ldots, y_L),$$

where $y_i = \theta s_i + v_i + n_i$, $i = 1, \ldots, L$; θ is a numerical parameter, $s_i = s_i(t)$, $v_i = \operatorname{Re} v(\lambda) \exp(j\omega t + j\vartheta_i)$ are the processes which describe the observed and interfering signals which are assumed to be narrow band with a mean frequency of ω, and $n_i = n_i(t)$ are processes of the white noise type with a correlation function

$$B_{ij}\delta(t-\tau) = M n_i(t) n_j(\tau).$$

The sources of the processes $S = \{s_i, i = 1, \ldots, L\}$, $V = \{v_i, i = 1, \ldots, L\}$ are described by the determined functions s(t) and v(λ). The random parameter $\lambda = \lambda(t)$ is assumed, a priori, to be given as a diffusion process which corresponds to a differential equation. The unknown parameter (continuous) $\vartheta = \{\vartheta_i, \quad i = 1, \ldots, L\}$ is determined by the shift in time for the interfering signal at the output to the elements in the antenna system. An a priori distribution P(ϑ) is given.

The optimum algorithm for the observation must be found, i.e., the discriminations for the hypothesis H_0: $\theta = 0$ and H_1: $\theta > 0$.

As is known, the ratio of the similarity of l(Y) is a sufficient statistic in problems of detection.

The determination of l(Y) will be based on the following auxiliary assertion. The ratio of the similarity l(Y,ϑ) for a known v allows us to write

$$l(Y,\vartheta)=l_0(Y)+l_1(Y-\vartheta S,\vartheta)-l_1(Y,\vartheta), \qquad (1)$$

where $$l_0(Y)=\vartheta\int_0^T(S^{\tau}B^{-1}Y-\frac{1}{2}\vartheta S^{\tau}B^{-1}S)\,dt \quad \text{— or-}$$

is the ratio of the similarity in the problem of detection for $Y=\vartheta S+N \quad N^{\tau}=(n_1, n_2, \ldots, n_L)$;

$$l_1(Y,\vartheta)=\int_0^t[\overline{V}^{\tau}(\vartheta_0\mid H_0)B^{-1}Y-$$

$$-1/2\,\overline{V}^{\tau}(\vartheta\mid H_0)B^{-1}\overline{V}(\vartheta\mid H_0)]\,dt. \qquad (1a)$$

is the ratio of the similarity for the problem of detecting the interfering signal for the observation Y = V + N,

$$l_1(Y-\vartheta S,\vartheta)=\int_0^T\overline{V}^{\tau}(\vartheta\mid H_1)B^{-1}(Y-\vartheta S)-$$

$$-\frac{1}{2}\overline{V}^{\tau}(\vartheta\mid H_1)B^{-1}\overline{V}(\vartheta\mid H_1)]\,dt, \qquad (1б)$$

where $\overline{V}(\vartheta\mid H_l)=M\{V\mid H_l,\ Y(\tau),\ \tau\leqslant t\},\ l=0,\ 1$ is the a posteriori mean value of the vector V.

Equation (1) follows from simple transformations of the Radon-Nikodim derivative for dimensions which correspond to the diffusion processes [4] Y = S + V + N, Y = V + N for a known ϑ. In the case which is of interest to us (ϑ unknown) the generalized ratio of the similarity [5]

$$l(Y)=\ln\int\exp l(Y,\vartheta)\,dP(\vartheta). \qquad (2)$$

is a sufficient statistic.

If Θ is a small parameter 1(Y) allows us to write

$$l(Y) = l_0(Y) - \vartheta\rho(Y) + 1/2\,\vartheta^2\rho^2(Y) + o(\vartheta^2), \qquad (3)$$

where

$$\rho(Y) = \frac{\left\langle \exp l_1(Y,\vartheta)\int_0^T S^\tau B^{-1}\overline{V}(\vartheta \mid H_0)\,dt \right\rangle_\vartheta}{\langle \exp l_1(Y,\vartheta)\rangle_\vartheta};$$

$\langle\cdot\rangle$ indicates averaging with respect to $P(\vartheta)$.

Equation (3) follows from the Taylor expansion for $\langle \exp l_1(Y - \vartheta S, \vartheta)\rangle_\vartheta$ at the point $\Theta = 0$ and for expression $\ln(1+x) = x + o(x)$.

The use of 1(Y), determined by means of expression (3) is related with the averaging in terms of ϑ. The integrand in (2) contains terms of the type

$$w_t(\vartheta) = \exp l_1(Y, \vartheta)\,dP(\vartheta), \qquad (4)$$

which is a nonstandardized a posteriori dimension for the random parameter ϑ.

Let us consider the asymptotic properties of the dimension $w_t(\vartheta)$ for $t \to \infty$.

If in the vicinity of ϑ_0 for the true value of the parameter

$$V(\vartheta) - V(\vartheta_0) = \frac{\partial V(\vartheta_0)}{\partial\vartheta}(\vartheta - \vartheta_0) + o(\mid \vartheta - \vartheta_0 \mid) \qquad (5)$$

or any $n < L - 1$ and $\vartheta_{0\,n+1} - \vartheta_{0\,n} < \pi$ there is

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T M \mid v_i^{\cdot}(\lambda) \mid^2 dt = \text{const}, \qquad (6)$$

then the following expression occurs

$$\ln w_t(\vartheta) = \ln w_t(\hat{\vartheta}) - \frac{1}{2}(\vartheta - \hat{\vartheta})^\tau Q(E + \gamma)(\vartheta - \hat{\vartheta}), \qquad (7)$$

in which $\hat{\vartheta}$, Q satisfy the system of stochastic equations

$$\frac{d\hat{\vartheta}}{dt} = Q^{-1}\,\frac{\partial \overline{V}(\hat{\vartheta})}{\partial \vartheta}\,B^{-1}(Y - \overline{V}(\hat{\vartheta})); \qquad (8a)$$

$$\frac{dQ}{dt} = \frac{\partial}{\partial \vartheta}\left[\frac{\partial \overline{V}(\hat{\vartheta})}{\partial \vartheta}\,B^{-1}(Y - \overline{V}(\hat{\vartheta}))\right]; \qquad (8б)$$

$$\overline{V}(\vartheta) \equiv \overline{V}(\vartheta \mid H_0); \quad \frac{\partial V(\vartheta)}{\partial \vartheta} = \left\{\frac{\partial}{\partial \vartheta} v_i \,\Big|_{\vartheta = \hat{\vartheta}}\right\}_{i,j=1,\dots,L};$$

$$Q = \{Q_{i,j}\}_{i,j=1,\dots,L};$$

E is a single matrix;

$$\gamma = \{\gamma_{ij}(t)\}_{i,j=1,\dots,L}, \ \lim_{t\to\infty} \vartheta_{ij}(t) \to 0.$$

In this case the solution to equations (8a, b) has the properties a) $Q_{ij} \to \infty$ c $P=1$ for $t \to \infty$;
b) $P\{\lim Q_{ij}/ct = 1\} = 1$, where $c = |v|^2 B_{ij}^{-1} \cos(\vartheta_{0i} - \vartheta_{0j})$;
c) the evaluation of $\hat{\vartheta}$ for the hypothesis H_0 is reduced to the true value of ϑ_0 in terms of the probability for $t \to \infty$.

The expression for the expansion (7) and for equations (8a, b) can be found from (1a) by using the Taylor expansion with respect to the point $\hat{\vartheta}$ and the well known method for obtaining equations for evaluating $\hat{\vartheta}$ and the matrix Q [7]. The use of known results [6, p. 120, 125] to analyze (8b) confirms the properties a and b. The validity of the evaluation of $\hat{\vartheta}$ follows from a study of the properties of the solution to the equations for $x = \hat{\vartheta} - \vartheta_0$ using the property b.

We note that system (8) determines the structure of the asymptotic for the optimum block for evaluating the unknown parameter v. For a narrow band FM interfering signals the recurrent equation for evaluating $\hat{\vartheta}$ (8) can be rewritten

in the form $B_{dg} = (B_{kk}, k = 1, \ldots, L)$

$$\frac{d\hat{\vartheta}_r}{dt} = -\sum_{k}^{L} Q_{rk}^{-1} B_{kk}^{-1} \bar{v}(\lambda) y_k \sin(\omega t + \hat{\vartheta}_k). \quad (9)$$

The structural scheme for the block for evaluating $\hat{\vartheta}$, which corresponds to (9) for the FM interfering signal and the diagonal matrix B is shown in Fig. 1. The evaluating block is multichannel at the input to the system. The following designations are used in the Figure FV - block for controlling phase rotaters, G - generator which is the block for evaluating the interfering signal $\mathrm{Re}\, \bar{v}(\lambda) \exp j\omega t$; $Q^{-1}x$ is the block for rotating the matrix Q and multiplication by the matrix Q^{-1} to the left, the double arrows indicate the transmission circuits for the vector processes.

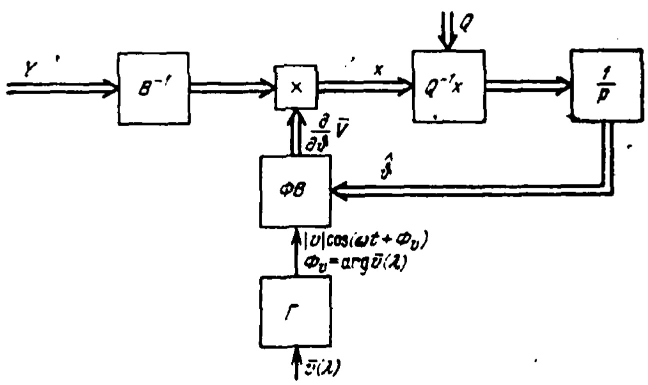

Fig. 1

Taking the asymptotic behavior of Q into consideration, it makes sense, for simplification, to use the asymptotically equivalent function

$$Q_{kk} = \rho_k t, \quad k = 1, \ldots, L,$$

in place of Q_{kk}, where $\rho_k = |\bar{v}(\lambda)|^2 B_{kk}^{-1}$ is the interference/noise ratio in the kth channel. In this case,

the evaluating block is described by the equation

$$\frac{d\hat{\vartheta}_k}{dt} = -\frac{1}{\rho_k t} \overline{\sum_{l=1}^{L} |\bar{v}(\lambda)| B_{ll}^{-1} y_l \times \times \sin(\omega t + \Phi_v + \hat{\vartheta}_l)}, \quad (10)$$

where $\Phi_v = \arg \bar{v}(\lambda)$; and $\overline{\sin(\cdot)}$ indicates the averaging in terms of the a posteriori dimension of the parameter λ.

The evaluating scheme which corresponds to (10) is shown in Fig. 2.

II. The evaluation $\bar{V}(\vartheta|H_0)$, which corresponds to the problem of filtering an a priori given modulating function v(λ) from the observation of $y_l = v_l + n_l$, $l = 1, \ldots, L$, for a fixed value of ϑ enters into equation (3).

Let us designate $\lambda \equiv \Phi_v$; $\hat{\lambda} = M\{\lambda \mid Y, H_0\}$ and let us fix the parameter λ by the equation

$$e\lambda/dt = a(t)\lambda + b(t)\xi(t), \quad (11)$$

where ξ(t) is the δ-correlated process with a zero mean and $M\xi(t)n = 0$.

By using the method of quasilinear optimum filtration [7] to solve the problem, we can write the structure of the block for evaluating the function v(λ) as a system of equations (for diagonal matrix b):

$$d\hat{\lambda}/dt = a(t)\hat{\lambda} + K(t)\sum_{m}^{L} B_{mm}^{-1} (\partial/\partial\lambda)\, \bar{v}_m y_m;$$

$$dK(t)/dt = 2a(t)K(t) + b^2(t) -$$

$$-K^2(t)\frac{\partial}{\partial\lambda}\left[\frac{\partial \bar{V}^T}{\partial\lambda} B^{-1}(Y - \bar{V})\right], \quad (12)$$

where $\bar{v}_m = v_m(\hat{\lambda}) = \mathrm{Re}\, v(\hat{\lambda}) \exp(j\omega t + j\vartheta_m)$; $K(t)$— is the a posteriori dispersion in the evaluation. In this case, as before,

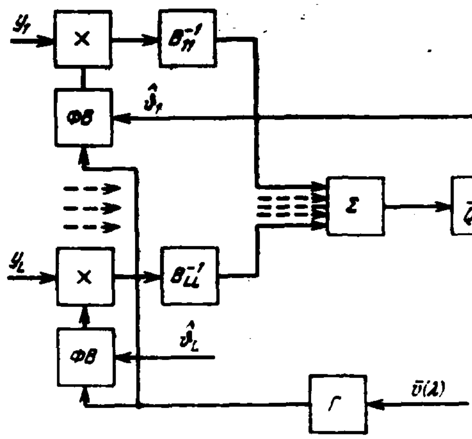

Fig. 2

the time of the delay in the interfering signal at the aperture is assumed to be much less than the interval of correlation for the process λ.

Equation (12) describes the L-channel phase system with a common controlling generator. The spatial treatment of the observation is reduced to the multiplication of the signal y_m, which is received in each channel by the supporting signal and the weighted summation of the result.

Let us determine the effect of the spatial treatment on the accuracy of determining λ. For simplicity in the description we will limit ourselves to a study of a linear aperture and stationary values of $K(t)$ which corresponds to the problem with $a(t)$ = const, $b(t)$ = const. The expression

which determines $K = \lim_{t\to\infty} K(t)$:

$$K = \frac{1}{\rho_v G(\Delta)} \left(\sqrt{1 + \frac{b^2}{(a)} \rho_v G(\Delta)} - 1 \right), \qquad (13)$$

follows from (12), where $\rho_v = |v|^2/2|a|B_{11}$ is the interference/noise ratio in the interference band

$$G(\Delta) = \cos\left[(L-1)\frac{d\Delta}{\Lambda}\right] \frac{\sin(\pi dL\Delta/\Lambda)}{\sin(\pi d\Delta/\Lambda)} \qquad (14)$$

is a function which characterizes the directional nature of the system for treating the observations (the directional diagram (DN) for the block for evaluating λ); $\Delta = (\vartheta, 2\pi)(\Lambda/d)$ is the generalized, angular direction [8] at the source of the interfering signal with respect to the aperture, d is the distance between the elements of the antenna grating, and Λ is the wavelength.

As we can see from (13), the orientation of the DN on the sources greatly improves the evaluation accuracy. In the case for which optimum evaluation is used without the spatial treatment G(Δ) in (13) is determined for the orientation of the DN on the wanted source and it can greatly lower the quality of the evaluation.

III. The properties which have been given for the functions $w_l(\vartheta), l(Y)$ can be used to convert to a description of the treatment algorithm, as determined by equation (3) in which

$$\rho(Y) = \int_0^T S^{\tau} B^{-1} \overline{V}(\hat{\vartheta}/H_0)\, dt. \qquad (15)$$

Thus, the structure of the receiver is determined by the expressions in (3), (12), and (8). The next problem

is to evaluate the quality of the receiver's operation.

IV. It is apparent that the quality of the observing procedure is determined by the properties of the statistics for l(Y). Usually finding the distribution function for l(Y) is a difficult problem. Therefore, for a large observation time the asymptotic properties of l(Y) may prove useful.

Let the process λ be given by (11), v_i is the FM signal and the following conditions are fulfilled

$$\frac{1}{t}\int_0^t a(s)\,ds \underset{t\to\infty}{\to} -c, \quad c>0;$$
$$\frac{1}{t}\int_0^t K^2(s)\,ds \to \text{const}, \quad t\to\infty. \tag{16}$$

Then the distribution for l(Y), which is asymptotic with respect to t is determined as the distribution for the random quantity $\mu = \mu_1+\mu_2$, in which $\mu_1 \equiv \mu_1(t)$ и $\mu_2 \equiv \mu_2(t)$ have the distributions shown below

for μ_1

$$\rho_s^{-1/2}(\mu_1-\overline{\mu}_1)\sim N(0,\,1), \quad t\to\infty; \tag{17}$$
$$\overline{\mu}_1=\begin{cases}-0{,}5\rho_s & \text{при } H_0,\\ 0{,}5\rho_s & \text{при } H_1;\end{cases}$$

for μ_2

$$w(x)=(2\pi\psi_2(t)\,x)^{-1/2}\exp[-x/2\psi_2(t)], \quad x>0, \tag{18}$$

where $\rho_s=\frac{\theta^2}{2}\sum_{kl}^{L}B_{kl}^{-1}$ — is the signal/noise ratio

$$\psi_2(t)=(\theta\,|\,v\,|\,/2B_{11})^2G^2(\Delta_v)D(r_t);$$

$$D(r_t)=M(r_t-Mr_t)^2; \quad r_t=\frac{1}{|\,v\,|}\int_0^t s(t)\overline{v}(\lambda)\,dt; \tag{19}$$

$G(\Delta_v)=\sum_{k,l}^{L} a_{kl}\cos l\hat{\vartheta}_1$ is the value of DN in the direction of the source of the interfering signal $a_{kl}=B_{11}/B_{kl}$.

The proof of the asymptotic properties of l(Y) is based on getting an expression which is valid for large enough values of t,

$$l(Y)=\theta\int_0^t S^T B^{-1}(Y-\overline{V}(\vartheta_0/H_1)-\theta S)\,dt+$$

$$+\frac{1}{2}\theta^2\int_0^t S^T B^{-1} S\,dt-\eta_1+\eta_2, \tag{20}$$

where $\eta_1\equiv\eta_1(t)=\theta\int_0^t S^T B^{-1}\times$

$$\times\left[(\hat{\vartheta}-\vartheta_0)\frac{\partial}{\partial\vartheta}\overline{V}(\vartheta_0\mid H_0)\right]dt,$$

$$\eta_2\equiv\eta_2(t)=\theta^2\rho^2(Y)/2,$$

in which the first two terms, obviously, form the normal, random quantity (17) and the functionals η_1, η_2 in the broad cases for the ergodicity of the processes which determine them because of the central, limiting theorem [9], are asymptotic normals:

$$[\psi_i(t)]^{-1/2}\eta_i\sim N(0,1),\quad t\to\infty,\quad i=1,2,$$

where $\psi_1(t)=O(t^{-1+\delta}),\ 0<\delta<1$; $\psi_2(t)$ is determined from (19).

V. In conclusion let us point out that the structure of the algorithm for the space-time treatment in the form of (1) is obtained for a wider class of signals in contrast to [8]. The conversion to the algorithm in the form of (3) can be used to determined the structure of the multi-channel receiver for a weak signal with respect to the interfering signal. In this case it is no longer necessary to form estimates of the parameters of the interfering signals

for the two hypotheses. An analogous approach to the synthesis of multichannel treatment can also be developed for the more complicated cases, for example, with any other indefiniteness in describing the interfering signal [8].

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Effect of Noises on Transition Processes in Systems for the Automatic Phase Tuning of Frequency

V.N. Kuleshov and N.N. Udalov

The effect of wide band noise on the time for establishing synchronized conditions and its reliability in astatic and FAPCh systems with an integrating filter was studied. Relations of the average time for establishing synchronized conditions, the dispersions of this time and the reliability of the synchronous conditions for an astatic FAPCh system to the noise level in a range of parameters which are of practical interest, were obtained.

Introduction. Systems for the automatic phase tuning of frequencies (FAPCH) are often used to separate a signal from the noise [1-3]. The selection of the filter in the feedback circuit determines the filtering of the interference under synchronized conditions. The effect of the interferences, which act at the input of such a system, on its dynamic characteristics, e.g., time of the transition process, capture band, and reliability of the synchronized conditions is of great interest.

The purpose of this work was to study the effect of wide band noise on a FAPCh system under transition conditions and also on its behavior under synchronized conditions.

Approach to the calculation of the mean time of establishing synchronized conditions in stochastic FAPCh systems. The problem of establishing synchronized conditions in FAPCh systems during the action of interferences is not completely solved at the present time. There are different methods

of evaluating the time for establishing the transition processes in FAPCh system in the presence of noise [5-8].

R.L. Stratonovich, R.Z. Khasminskiy, and Yu. A. Mitropolskiy developed a method of averaging for stochastic second order systems. This method was used to get a one-dimensional, approximate Fokker-Planck equation (FP) for the energy which characterizes motion in the FAPCh. Consequently, the solution to the problem of the mean time for reaching the limit for the first order Markov process can be used to evaluate the time at which the FAPCh system becomes synchronized [10]. We will discuss the approach to the solution of this problem.

The nonstationary FP equation for the probability density for the energy A in the FAPCh system W(A), in the general case, has the form [9]

$$\dot{W}(A)=\frac{\varepsilon}{D}\frac{\partial^2}{\partial A^2}\{\Phi_2(A)\,W(A)\}-$$
$$-\varepsilon\frac{\partial}{\partial A}\{\Phi_1(A)W(A)\}. \qquad (1)$$

where

$$A=\frac{(\dot{\varphi})^2}{2}+\int_0^{\varphi}F(\varphi)\,d\varphi$$

is the energy stored in the FAPCh system, φ is the phase difference for the oscillation of the tuned generator and the signal, $F(\varphi)$ is the normalized characteristic of the phase detector. The dot indicates differentiation with respect to the dimensionless time $\hat{t}=\varepsilon\Omega_c t$; Ω_c is the band of synchronism for the FAPCh system.

The coefficients $\Phi_1(A), \Phi_2(A), \varepsilon$ and D^{-1} are determined by the parameters of the system and the intensity of the noise. The condition for the sufficient accuracy for the approximate equation (1) is $\varepsilon \ll 1$. The stochastic equation for the system which corresponds to equation (1) has the form

$$\dot{A} = \varepsilon \Phi_1(A) + \sqrt{\frac{2\varepsilon}{D} \Phi_2(A)}\, \xi(\hat{t}),$$

where $\xi(\hat{t})$ is the delta-correlated process of unit intensity, for a zero intensity for the noise $D=\infty$, and this equation is converted into the equations for the energy found in [11], and the coefficient $\varepsilon\Phi_1(A)=f(A)$ determines the rate of decrease in the energy A in the absence of noise. The values of $\Phi_1(A), \Phi_2(A), \varepsilon,$ and D will be calculated in analyzing concrete systems.

Let us consider that the synchronized conditions are established if the energy A, which at the initial moment was equal to A_0 reaches a value Δ which is characteristic for tbe steady state conditions for the system. The problem of determining Δ will be discussed specifically.

The equation for the mean time $\hat{t}(A_{rp}/A_0)$ for the process A(t) reaching the limit A_{rp} was found in [10]. In our case it has the form

$$\frac{\varepsilon}{D} \Phi_2(A_0) \frac{d^2 \hat{t}(A_{rp}/A_0)}{dA_0^2} + \varepsilon \Phi_1(A_0) \frac{d\hat{t}(A_{rp}/A_0)}{dA_0} + 1 = 0. \quad (2)$$

From this we have, for the upper limit $A_{rp} = A_s > A_0$

$$\hat{t}(A_s/A_0) = \int_{A_0}^{A_s} \left\{ \int_0^z \frac{D}{\varepsilon \Phi_2(y)} e^{\psi(y)} dy \right\} e^{-\psi(z)} dz, \quad (3)$$

and for the lower limit $A_{гр} = A_{н} < A_0$

$$\hat{t}\,(A_{н}/A_0) = \int\limits_{A_{н}}^{A_0} \left\{ \int\limits_{z}^{\infty} \frac{D}{\varepsilon\, \Phi_2(y)}\, e^{\varphi(y)}\, dy \right\} e^{-\varphi(z)}\, dz, \qquad (4)$$

$$\text{where } \varphi(z) = \int\limits_{0}^{z} \frac{D\Phi_1(y)}{\Phi_2(y)}\, dy.$$

Solutions (3) and (4) satisfy the zero limit for the equation [10]

$$\hat{t}(A_{в}/A_0 = A_{в}) = 0; \quad \hat{t}(A_{н}/A_0 = A_{н}) = 0. \qquad (5)$$

The quantity which is inverse to the first integral in equation (2) $dA_0/d\hat{t}$, characterizes the mean statistical rate of change for the energy A for the action of noise and, in the absence of noise, it is equal to the rate of change for A.

Let us introduce the average time $\bar{t}$ for the process A(t) reaching the limit for some probability density for the initial energy $W_0(A_0)$:

$$\bar{t} = \int\limits_{A_1}^{A_2} t\,(A_0)\, W_0\,(A_0)\, dA_0, \quad A_1 \leqslant A_0 \leqslant A_2.$$

If $A_{н} = \Delta$, $A_{в} = \infty$, then the average time for the process A(t) to reach the limit $\Delta < A_0$ characterizes the duration of the transition process in the FAPCh system in the presence of noises. However, after the boundary has been reached the coordinate can leave the range of synchronized conditions. For the complete determination of the effect of noise on the dynamic properties the reliability of the synchronized conditions must be evaluated.

Evaluation of the reliability of the synchronized conditions in stochastic FAPCh systems. Let us select the position of the boundary Δ. We will assume that Δ is equal to the mathematically expected energy A(t) under stationary conditions $\Delta = \langle A_{ст}(t)\rangle$. The value of $\langle A_{ст}(t)\rangle$ is proportional to the sum of the mean squares of the coordinates of the system which determine the precision of the synchronization. The quantity $\langle A_{ст}(t)\rangle$ is determined from the solution of the stationary FP equation [10]

$$\langle A_{ст}(t)\rangle = \int_0^\infty A W_{ст}(A)\, dA, \qquad (6)$$

where
$$W_{ст}(A) = C\frac{D}{2\varepsilon\Phi_2(A)}\exp\left[\int_0^A \frac{D\Phi_1(A)}{\Phi_2(A)}\,dA\right],$$

and the constant C is determined from the normalizing conditions for $\int_0^\infty W_{ст}(A)\, dA = 1$. After the boundary Δ has been reached after a time $\hat{t}(\Delta/A_0)$ the time must be determined that the system remains under synchronized conditions. We know that if the energy A(t) exceeds 2 (leaving the region of rigorous synchronization) causes a transition processes with respect to the frequency which can cause a prolonged interruption in the system's operation. Therefore, the time for the reliable operation of the system will be considered to be the time during which the coordinate for A(t) moves along a trajectory which begins at $A = \Delta$ and which lies entirely in the range of rigorous synchronization $A < A_B = 2$. The mean value for the duration of this interval will be called the mean time of reliable operation for the system $\hat{t}(2/\Delta)$. Reaching A = 2 does not always mean that there is a phase jump of 2π for the origin of a long transition process. The point can return from A = 2 into the range of synchronized conditions without exceeding $A_B = 2$. The

time $\hat{t}(\Delta/2)$ for the return of the trajectory from the level $A_0 = 2$ to the level Δ is an additional characteristic for the operation of the sytem under synchronized conditions.

We will assume that the system is under synchronized conditions if its movement takes place along a trajectory which begins at the point $A_0 = \Delta$ (point 1, Fig. 1a) and ends at the point $A = 2$ (point 2) and that it operates under asynchronized conditions if its motion follows the trajectory which starts at the point $A = 2$ (point 2, Fig. 1a) and ends at the point $A = \Delta$ (point 1).

Thus, we will introduce a random process with two states (Fig. 1b) which reflect the synchronized and asynchronized conditions to describe the reliability of the synchronization in the system. The probability for the synchronized P_c and asynchronized P_a conditions are determined by the expressions

$$P_c = \frac{\hat{t}(2/\Delta)}{\hat{t}(2/\Delta) + \hat{t}(\Delta/2)};$$

$$P_a = \frac{\hat{t}(\Delta/2)}{\hat{t}(\Delta/2) + \hat{t}(2/\Delta)}. \qquad (7)$$

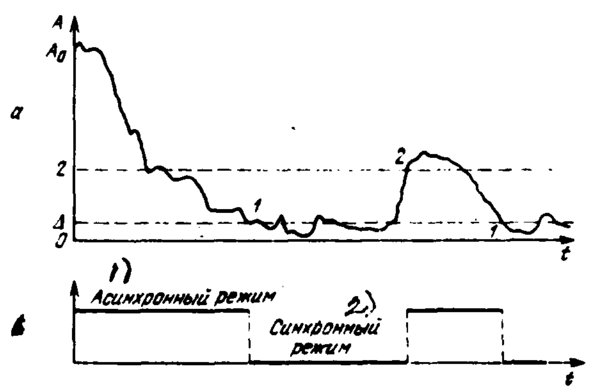

Fig. 1
1) Asynchronized conditions
2) synchronized conditions

Thus, by knowing the values of $\hat{t}(\Delta/2)$ and $\hat{t}(2/\Delta)$ we can evaluate the probability that the system will operate under synchronized conditions P_c. The length of time that the system will remain in the synchronized state is determined by the mean time required for the Markov process $A(\bar{t})$ which is moving from the level A_0 to reach the boundary $\Delta = \langle A_{cr}(t) \rangle$. This time $\hat{t}(\Delta/A_0)$ is used, then, to evaluate the duration of the transition processes in a system for a given probability for synchronized conditions P_c.

Evaluation of the effect of noise on the dynamic characteristics of an astatic FAPCh system. We will carry out the calculation of the mean time required to reach synchronized conditions in an astatic FAPCh system and for the reliability of its operation under these conditions.

The operating transmission coefficient for the filter in the feedback circuit for such a system has the form $K(p) = (1 + pT_1)/pT_2$, where $p = d/dt$, T_1, T_2 - are the time constants for the filter. Assuming that the characteristic of the phase detector is sinusoidal and introducing the normalization parameters for the filter [11] $b_1 = \Omega_c T_1$ and $a_1 = \Omega_c T_2$ (Ω_c is the synchronization band for the FAPCh without a filter), we will find the coefficients of ε & $\Phi_1(A)$, $\Phi_2(A)$ in the asymptotic case [9] ($\varepsilon = 1/\sqrt{a_1}$):

$$\Phi_1(A) = \begin{cases} -(4b_1/3K_1)\left\{K_1\left(1 - \dfrac{A}{2}\right) + (A-1)E_1\right\} + \dfrac{1}{D}, & A \leqslant 2; \\ -(4b_1/3K_2)\sqrt{A/2}\{(A-1)E_2 - \\ \quad -(A-2)K_2\} + \dfrac{1}{D}, & A > 2; \end{cases} \tag{8}$$

$$\Phi_3(A)=\begin{cases}(4/K_1)\left\{E_1+\left(1-\frac{A}{2}\right)K_1\right\}, & A<2;\\ 2A\,(E_2/K_2), & A>2;\end{cases} \qquad (9)$$

where $E_1=E(\sqrt{A/2}),\ K_1=K(\sqrt{A/2}),$
$E_2=E(\sqrt{2/A}),\ K_2=K(\sqrt{2/A})$ —

are the total elliptical integrals of the first and second order [12].

The value of the parameter D which characterizes the intensity of the noise is determined by the formula [9]:

$$D=2\Omega_c/K;$$
$$K=\sigma_1^2\frac{\Omega_c^2}{U_c^2}\int_0^\infty R(\tau)\,d\tau=\frac{\Omega_c^2}{U_c^2}N_0, \qquad (10)$$

where $R(\tau)$ is the normalized autocorrelation function for the input noize ξ (t) brought to the output of the FD ($\overline{\xi\xi_\tau}=\sigma_1^2 R(\tau)$); , N_0 is the spectral density of the input noise at the central frequency of the tuned generator, τ_k is the correlation time for ξ (t). It is assumed to be the smallest of the time constants for the system. The parameter D is the ratio of the signal and noise power in the Ω_c band [4].

A computer must be used to calculate the values of $\hat{t}(\Delta/A_0),\ \hat{t}(\Delta/2),\ \hat{t}(2/\Delta)$ by means of equations (3) and (4) taking (8) and (9) into account since the integrals which enter into these expressions cannot be expressed in known functions. However, it appeared that a qualitatively true and quantitatively rather accurate picture is found for

the effect of noise on the system's dynamics for the approximations (8) and (9) of their expansion into a series for $A \ll 1$ and $A \gg 2$, respectively.

In Fig. 2 the dependence $\Phi_1(A)$ in the absence of noise is shown by the dotted line. Its approximation is shown by the dependence $\Phi_1(A)$ for $Db_1 = \infty$. A comparison of these shows that for large initial energies A_0 the accuracy for determining the duration of the transition processes by means of such an approximation is satisfactory. Therefore, in place of (8) and (9) we write

$$\Phi_1(A) \approx \begin{cases} -b_1 A + 1/D, & A < 2; \\ -b_1/2 + 1/D, & A > 2; \end{cases} \quad \Phi_2(A) \approx \begin{cases} A, & A \leqslant 2; \\ 2A, & A > 2. \end{cases} \quad (11)$$

The inverse value of the mean statistical rate of decrease for the energy A from the level A_0 in the direction $A = 2 < A_0$ according to (4) will be determined by the expression

$$\frac{d\hat{t}(2/A_0)}{dA_0} = \frac{-1}{\varepsilon \Phi_1(A_0)} = \frac{1}{\varepsilon (b_1/2 - 1/D)}, \quad A_0 \geqslant 2. \quad (12)$$

As we can see from (12), the rate of decrease in the system's energy during the transition process for frequency, decreases with an increase in the noise.

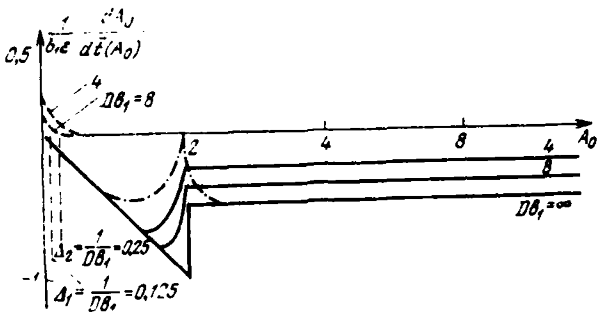

Fig. 2

The quanitity $d\hat{t}(\Delta/A_0)/dA_0$ for $A_0 < 2$ is found from (4) taking (11) into account:

$$\frac{d\hat{t}(\Delta/A_0)}{dA_0} = \frac{1}{\varepsilon A_0}\left[1 - e^{-Db_1(2-A_0)}\left(1 - \frac{1D}{Db_1 - 2}\right)\right] A_0 \leqslant 2. \tag{13}$$

The relationships for $-dA_0/d\hat{t}(A_0)$, calculated by (12) and (13) for different values of D are shown in Fig. 2 by the solid lines.

The expression for the mean statistical velocity, which determines the increase in the system's energy for $A_0 \leqslant 2$ and $A_B \leqslant 2$, calculated by means of equation (3), has the form

$$\frac{dA_0}{d\hat{t}(A_B/A_0)} = \varepsilon b_1 A_0 (e^{Db_1A_0} - 1)^{-1}, \quad A_0 \leqslant 2. \tag{14}$$

The value of this rate determines the time in which the mean energy of the system reaches the value $A_B \leqslant 2$. This dependence is shown in Fig. 2 by the dotted line. As the noise increases the rate of increase for the energy from $A_0 = \Delta$, increases.

Let us consider stationary conditions. From (6) the mean value of the energy is $\langle A_{ст}(t)\rangle = 1/Db_1 = \Delta$ for low noises $Db_1 \gg 1$ (Fig. 2). The lower limit for $A_H = \Delta$ has been determined. By carrying out the integration in (12), (13), and (14) and taking into consideration the normalization $(\hat{t} = (\Omega_c/\sqrt{a_1})t)$, we find a formula for the time needed to establish synchronous conditions $t(\Delta/A_0) = t(2/A_0) + t(\Delta/2)$ and the value of $t(2/\Delta)$:

$$t(2/A_0) = \frac{a_1}{\Omega_c} \frac{2D}{Db_1 - 2}(A_0 - 2); \quad (15)$$

$$t(\Delta/2) = \frac{a_1}{\Omega_c b_1}\Big[\ln Db_1 - e^{-2Db_1}\Big(1 - \frac{4Db_1}{Db_1 - 2}\Big)\times$$

$$\times(\mathrm{Ei}^*(2Db_1) - \mathrm{Ei}^*(1))\Big]; \quad (16)$$

$$t(2/\Delta) = \frac{a_1}{\Omega_c b_1}[\mathrm{Ei}^*(2Db_1) - \mathrm{Ei}^*(1) - \ln 2Db_1], \quad (17)$$

where Ei* is the integral exponential function [12]. The average time for the transition process with respect to the frequency $\bar{t}_{f\text{ш}}$ for switching on a system with a random initial phase difference [13] is determined by the quantity $t(2/A_0)$ and the detuning γ if, in place of A_0 in Equation (15) we use its mean value $\bar{A_0} = (a_1\gamma^2/2) + 1$ [13]:

$$\bar{t}_{f_\text{ш}} = \frac{a_1}{\Omega_c b_1}\left(\frac{2Db_1}{Db_1 - 2}\right)\left(\frac{a_1\gamma^2}{2} - 1\right), \quad a_1\gamma^2/2 > 1. \quad (18)$$

The dependence of the ratios for the transition processes with respect to frequency in an astatic FAPCh system are given in Fig. 3a as a function of Db_1 for the action of a wide band noise on its input and without it ($Db_1 = \infty$). The value of this ratio, from Equation (18) is determined by the formula

$$\bar{t}_{f_\text{ш}}/\bar{t}_f = Db_1/(Db_1 - 2). \quad (19)$$

There is no apparent delay of the transition processes with respect to the frequency in such a system because of the noise $(\bar{t}_{f_\text{ш}}/\bar{t}_f < 1.1)$ or a signal/noise ratio of $q > 5$ in the system's band (Fig. 3a). The ratio q is determined according to [4] as the ratio of the signal's amplitude U_c to the mean square voltage for the noise $\sigma_\text{ш}$: $q = U_c/\sigma_\text{ш} = \sqrt{Db_1}$

In this case $P_c/P_ш = q^2/2$. The condition that the noise has little effect on the time for the transition processes in the astatic system $q > 5$ corresponds to $P_c/P_ш > 12.5$ in the system's band. The quantity $\bar{t}_{f_ш}/t_f$ in this case does not depend on the initial detuning γ. The value of the dispersion in the time for the transition process with respect to frequency $\sigma^2_{t_{f_ш}}$ can be determined from the following formula which was also obtained by the method in [14]:

$$\sigma^2_{t_{f_ш}} = \frac{4a_1^2}{\Omega_c^2 b_1^2}\left(\frac{Db_1}{Db_1-2}\right)^2 \cdot \frac{2}{Db_1-2} \times$$
$$\times\left[\left(\frac{a_1\gamma^2}{2}\right) + 2\left(\frac{a_1\gamma^2}{2}\right) - 1\right] \quad a_1\gamma^2/2 \geq 1. \qquad (20)$$

As might be expected, the quantity $\sigma^2_{t_{f_ш}}$ depends on the initial detuning (initial energy) and it increases as the detuning increases. The dependences for $\sqrt{\sigma^2_{t_{f_ш}}}/\bar{t}_{f_ш}$ on the intensity of the noise are given in Fig. 3b for two values of the generalized initial detuning $a_1\gamma^2/2$. We can see from the figure than an increase in the intensity of the noise results in an increase in $\sqrt{\sigma^2_{t_{f_ш}}}/\bar{t}_{f_ш}$.

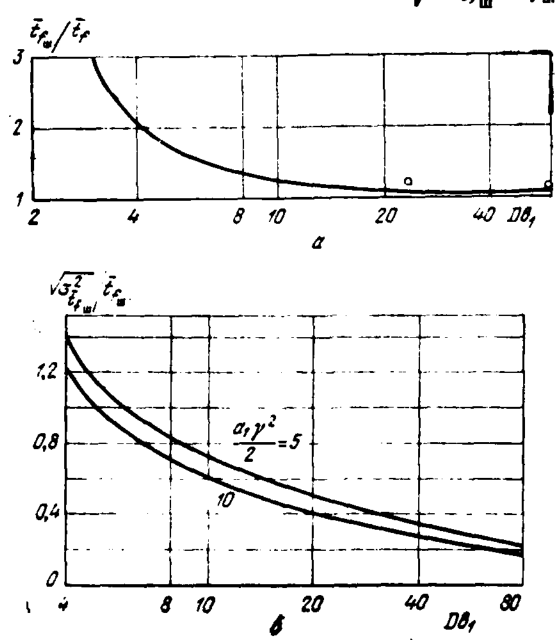

Fig. 3

The dependence of the average time $\bar{t}$ for establishing synchronous conditions in the system, calculated by equations (15) and (16) on the initial detuning is shown in Fig. 4 for different intensities of the noise at the input to the system. For small detunings, when the average time for establishing synchronous conditions is determined mainly by the transition process with respect to the phase (16), $\bar{t}$ decreases with an increase in noise and is less than the average time for the transition process in the absence of noise ($Db_1 = \infty$). This is explained by the fact that as the intensity of the noise increases the boundary $\Delta = 1/Db_1$, which determines the appearance of the synchronous conditions, shifts to the right (see Fig. 2), and the average time required for all of the trajectories to reach this boundary becomes less.

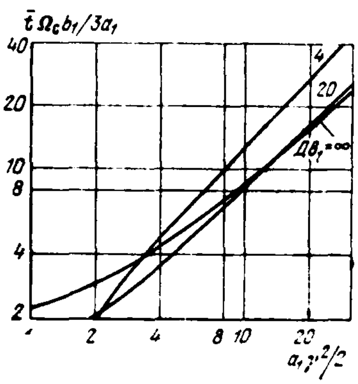

Fig. 4

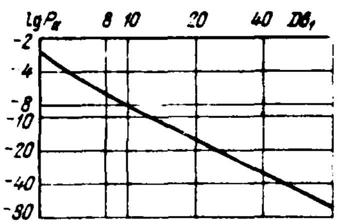

Рис. 5

Fig. 5

By using (16) and (17) we will calculate the probability P_c (7), which determines the reliability of the system's operation under synchronous conditions. For $Db_1 > 3$ the formula for calculating P_c has the form

$$P_c \approx 1 - \frac{3Db_1 + 2}{Db_1 - 2} e^{-2Db_1}. \qquad (21)$$

The dependence for $\lg \bar{P}_a = \lg(1 - P_c)$ on the intensity of the noise and the system's parameters is given in Fig. 5.

We can see that for q = 5, $P_a < 10^{-10}$ in the system's band. In this case, the average time for the reliable operation of the system $t(2/\Delta)$ without phase clicks is evaluated by equation (17).

We note that for $b_1^2/a_1 \gg 1$, by using an analogous approach in conjunction with the quasistationary method [9], we can get the following expression for the average time for the transition process with respect to frequency:

$$\bar{t}_{f_{ш}} = \frac{a_1}{\Omega_c b_1}\left(\frac{2Db_1}{Db_1-2}\right)\left(\frac{a_1\gamma^2}{2}-\frac{a_1 m^2}{2}\right), \quad m=\frac{b_1}{a_1}.$$

By comparing it with (18) we see that the effect of the noise on the value $\bar{t}_{f_{ш}}$ over the entire range of parameters for the astatic FAPCh system is the same. Therefore, the relationship for $\bar{t}_{f_{ш}}/\bar{t}_f$ (see Fig. 3a) for the asymptotic parameters $(b_1/\sqrt{a_1} \ll 1)$ is valid in the quasistationary case also. However, in the general case [4] $q = \sqrt{Db_1/(1+mb_1)}$. in the system's band. Therefore, the region $Db_1 > 20$ corresponds to the region $q > \sqrt{20/(1+mb_1)}$, in which the effect of the noise on the time for the transition processes can be ignored in astatic systems.

We can show, using an analogous approach, that in an FAPCh system with an integrating filter, the time for the transition processes decreases slightly with an increase in the noise. The qualitative difference for the effect of the noise on this system is probably associated with the fact that in the astatic FAPCh system there are wide increment zones on the phase plane in which, because of the noise, the accumulation of energy is more intense than its scattering in the decrement zones.

Conclusions: Calculation of the dynamic characteristics of stochastic FAPCh systems led to the following conclusions: a wide band noise leads to a delay in the average time for establishing synchronized conditions in systems with significant increment zones on the phase plane, i.e., in such systems in which attenuation during the period of beats changes sign (astatic system, FAPCh system with PIF), and hardly changes (somewhat decreases) the average time for establishing synchronous conditions in FAPCh systems in which the entire phase plane corresponds to a decrement zone (FAPCh with IF).

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SECTION II

PROBLEMS OF RECEIVING AND SYNCHRONIZING DISCRETE PM SIGNALS

Demodulation of PM Signals Using Digital Logical Elements

V.L. Banket and V.N. Batrakov

Systems with phase manipulation (PM) have a high freedom from interference and are being used ever more widely for the transmission of discrete communications. The conversion to multiple PM means that the channel's frequency bands can be well used. In this case the simplest method and that which is closest to the optimum method for recording samples is single reading. In this case the filtering of the signal-interference sum which is received is done by a filter at the input to the phase detector. The use of analog methods for treatment of the PM signals for a high multiplicity of manipulation gives a marked complication of the demodulator circuit with rigid requirements for the accuracy of the device and the stability of the parameters for its individual units. In this article the possibilities are discussed of building demodulators for a single reading of the PM signal that is received using discrete logic elements.

In the general case the signal with m-fold phase manipulation can be represented in the following manner:

$$\cos(\omega t+\Delta\varphi k), \quad 0 \leq t \leq T, \qquad (1)$$

where $k=0, 1, 2, \ldots, 2^m-1$ — is the position number of the phase for the signal, $\Delta\varphi=2\pi/2^m$ is the phase shift for the signals for neighboring positions, $\omega=2\pi/T_н$ is the carrier frequency, $T_н$ is the period of the carrier frequency,

m is the manipulation multiple, 2^m is the number of phase positions for the signal, and T is the duration of the signal sample.

The signal (1) is demodulated in the usual way by remultiplying it in the remultipliers (phase detectors) on supporting oscillations of the type

$$\cos(\omega t+\Delta\varphi\, l+\Delta\varphi/2) \qquad (2)$$

and by integrating the result of the remultiplying over the period of the carrier's signal $T_{н}$. As a result we get an oscillation

$$\cos\Delta\varphi\,(k-l-1/2), \qquad (3)$$

where l is the number of the reception channel. In the general case, the shaping device, in separating the reference oscillation from the phase manipulation signal which is received, generates a reference oscillation with phase indefiniteness of the 2^m order

$$\cos(\omega t+\Delta\varphi\, l+\Delta\varphi/2+\Delta\varphi n). \qquad (4)$$

As a result of remultiplying signal (1) by the reference oscillation (4) and integration, we get

$$\cos\Delta\varphi\,(k-l-n-1/2). \qquad (5)$$

Here the number n = 0, ±1, ±2,...,$\pm 2^m-1$ determines the value of the discrepancy for the initial phases of the signal and the reference oscillation. Determining the signals in double reception channels for a single reading is carried out by strobing the result of the integration in the center

of the sample and determining the sign of the result of the strobing.

Let us consider the possibility of demodulating the PM signal on the basis of discrete logic elements. Let us introduce the term $\Delta\varphi l + \Delta\varphi n + \Delta\varphi/2$ with positive and negative signs into the argument for the signal (1)

$$\cos[\omega t + \Delta\varphi\, l + \Delta\varphi n + \Delta\varphi/2 + \Delta\varphi(k - l - n - 1/2)]. \quad (6)$$

Assuming that $\Delta\varphi = 2\pi/2^m = \omega T_н/2^m = \omega\,\Delta t,$ where $\Delta t = T_н/2^m,$ equation (6) can be presented in the following form:

$$\cos[\omega(t + \Delta t l + \Delta t n + \Delta t/2) + \Delta\varphi(k - l - n - 1/2)]. \quad (7)$$

By reading the signal (7) and, then signal (1) also at a moment of time which satisfies the condition

$$\omega(t + \Delta t l + \Delta t n + \Delta t/2) = 2\pi p, \quad (8)$$

where p is a whole, positive number, we find the resulting signal of the reading

$$\cos\Delta\varphi\,(k - l - n - 1/2), \quad (9)$$

which coincides with equation (5). From the condition in (8) it follows that the formula for the moments of time of the readings is

$$t_i = T_н\, p - \Delta t l - \Delta t n - \Delta t/2. \quad (10)$$

Thus, the signal with phase manipulation can be demodulated for a single reading by determining its polarity at the moments of time given by equation (10).

The device for determining the polarity of the signal at given moments of time can be constructed by using a bistable cell with a pulsed supply for the collector circuit (Fig. 1).

The first bistable cell (BC1) is used as a high frequency discriminator of the polarity and the second (BC2) is used as a memory device.

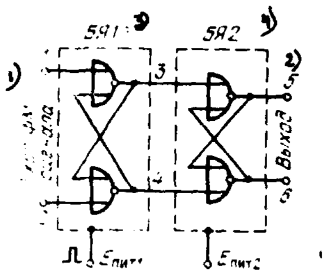

Fig. 1

1) input of PM signal,
2) output
3) BC1, 4) BC2.

By supplying a current pulse at the moment of reading the forward front, the bistable cell 1 becomes established in one of two stable states which depends on the polarity of the voltage difference at the inputs for cells 1 and 2. Cell 2 is used to remember the state of the bistable cell 1 in the interval between pulses. This cell is supplied from a source of constant voltage. Thus, at the outputs of BC2 one of two possible states will exist, depending on the polarity of the voltage difference at the inputs to 1 and 2 (Fig. 2). The polarity discriminator of this type operates over a wide range of transmission rates with a rather small width of the ambiguity band. The scheme of the device for determining the polarity is shown in Fig. 3. It is made up of Series 114 integrated microcircuits. Elements I and III are used in the flip-flop circuit. Elements II and IV are buffer elements for the conversion to the memory trigger (V and VII). A pulsed power voltage is supplied to elements I-IV. Element VI and VIII are used as buffers in the output circuits of the memory trigger. Devices based on other types of integrated microcircuits can be assembled in an analogous manner. Tests showed that the sensitivity (width of the ambiguous resolution zone) is equal to about 5 mV when Series 114 circuits are used for a frequency up to

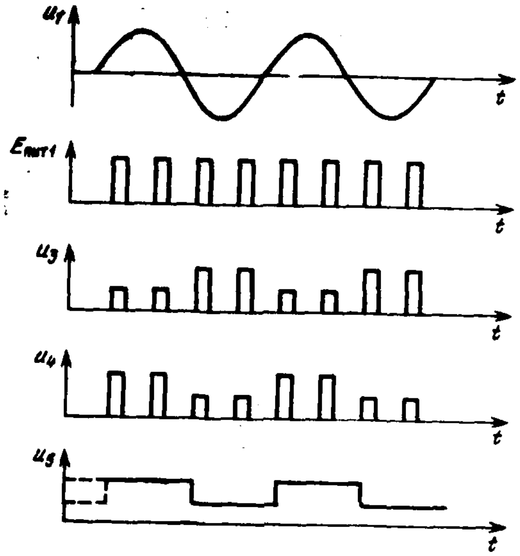

Fig. 2

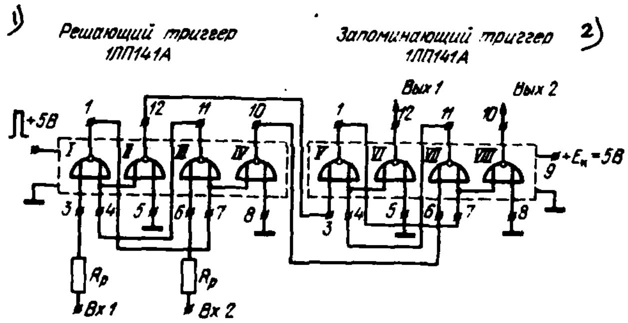

Fig. 3

1) resolving trigger, 2) memory trigger

100-200 kHz for the pulsed power supply. The frequency range can be greatly expanded if high speed logic systems are used. Thus, if Series 137 circuits are used a sensitivity of 2-3 mV is retained up to frequencies of the order of 5-8 mHz.

One of the important questions in building signal demodulators with PM is providing for an accurate establishment and maintenance of the phases for the supporting oscillations

during operation. It is possible, in using discrete methods of treating PM signals, to increase the accuracy of establishing the phases markedly. We will illustrate this for the demodulation of a signal with double phase modulation (m = 2).

As we know, if we use an analog method of demodulation the phase difference for the supporting oscillations of the demodulator with double PM should be equal to 90°. According to equation (10), the moments of time for the readings, using the given discrete method of demodulation are equal to:

in the first channel for $l=0$ $t_0 = T_H p - T_H/8$;

in the second channel for $l=1$ $t_1 = T_H p - T_H/4 - T_H/8$.

In this case it is assumed that $n = 0$, i.e., the phase ambiguity for the supporting oscillation is absent, $\Delta t = T_H/4$ and, in addition, the whole number p is chosen so that the product pT_H will be approximately equal to half of the duration of the sample. Thus, the demodulation of a signal with double phase manipulation can be accomplished by determining the polarity of the received signal at the moments of time which differ from one another by $T_H/4$. In this case, it is assumed that the demodulation in the first and second channels is accomplished by the first and second resolving device, respectively. In this case it is quite difficult to form two pulse power supply sequences which are shifted by exactly $T_H/4$. These difficulties can be avoided by using only one device for demodulating the signals in both channels. In this case a single, periodic pulse sequence is used with a period equal to $T_H/4$. In this case the device gives a resolution of the polarity of the received signal four times during the period of the carrier oscillation. Two of them, which differ from one another by a period of $T_H/4$ and are determined at moments of time t_0 and t_1, should

be used for the demodulation. The functional scheme for a demodulator of this type is given in Fig. 4. The selection of the resolutions is done by the commutator KOM which switches the output of the cell BC1 to the inputs to the memory cells BC2 (A and B). The operation of the cell BC1 and the commutator is controlled by a shaping circuit for the supporting oscillation (FSO). Since the receiving channels are separated by commutation of the resolution, the error in forming the commutation pulses is not reflected in the operation of the demodulator. The precision in establishing the counting interval, which is equal to $T_H/4$, is determined by the periodic sequence for the pulses and it can be made quite high.

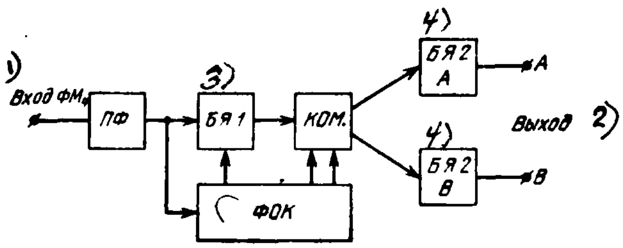

Fig. 4

1) input, 2) output, 3) BC1, 4) BC2

A demodulator, such as the one described, was made for receiving signals with double PM at a rate of 2 x 32 kbaud. The frequency of the input PM signal is 2000 kHz. Tests showed that the freedom from interference for the demodulator, built on the basis of discrete logic elements, differs very little from that for analog type demodulators. Thus, for a probability of error of 10^{-4} - 10^{-5} the energy losses are 1.1 - 1.5 dB which is determined mainly by the appearance of intersymbol interference during filtering of the PM signal. In addition, there is no low frequency filter in the demodulator based on discrete logic elements. It is set at the output to the phase detector to suppress the high frequency components which are obtained as the result of remultiplying.

Building a Discrete Communications System for the Transmission of Signals of Videotelephone Images

N.T. Petrovich, V.V. Pavlyuk,
E.N. Il'in and V.M. Voronovich

The problems of building a discrete communications system for transmitting videotelephone images are discussed. Methods are selected for manipulating the carrier, and the freedom from interference is calculated for these transmission methods. Discrete methods with continuous methods of modulation (AM and FM) are compared.

Communications technology in the past ten years has been characterized by the wide use of discrete signals for the transmission of analog (continuous) communications. The transmission of a videotelephone image by discrete signals has marked advantages over the traditional analogs: ability of regeneration at the retransmission points, which limits the accumulation of interferences, avoiding an excess image signal which contracts the volume of transmitted information by ten times and more, the use of highly reliable integrated and hybrid computer elements in building a discrete communications system, flexibility of the discrete communications system, convenience in coordinating with different communication channels, etc.

The videotelephone signal has a top frequency of 1 mHz and the signal/noise ratio must not be less than 30 dB. The following methods of modulation are possible: IKM, DIKM, and DM. For a satisfactory perception of the image for IKM it is sufficient to use 16 levels of quantization. Then the rate of transmission is 8 mbits/sec and the signal/noise ratio for quantization is 35 dB. For DIKM 8 levels of quantization are needed and the rate of transmission is 6 mbits/sec.

The freedom from interference for the channel is evaluated by the degree of visual discernibility of the distortions in the image as a function of the probability of error $P_{ош}$. For IKM (pulse code modulation), for $P_{ош} = 10^{-4}$ the demodulated image is discerned almost without distortion and for DIKM (delta pulse code modulation) $P_{ош} = 10^{-4}$ is not sufficient since interference is observed in the form of horizontal bands on the received image because of the accumulation of errors in the receiver. An acceptable image, from the standpoint of quality, is obtained with DIKM for $P_{ош} = 10^{-6}$.

For delta modulation (DM) a wider frequency band is needed for the transmission of videotelephone signals. It was shown in [2] that in order to get a signal/noise ratio of up to 40 dB the reading frequency must be twenty times greater than the top frequency of the videosignal, i.e., a transmission rate of the order of 20 mbits/sec is required for a videotelephone.

Thus, for channels with a limited energy potential it is recommended that IKM be used and for a large energy potential (cable or radio relay channels) DIKM should be used. It is not expedient to use delta modulation (according to preliminary studies) for the transmission of a videotelephone signal because of the relatively low efficiency although this method of conversion is technically much simpler than IKM and DIKM.

Experimental studies have shown that for a satisfactory quality of the image the usual 16-level IKM is not sufficient although the signal/noise ratio for the quantization exceeds 30 dB. This is due to the subjective properties for discerning the image by people (isolation of spurious shapes). It is

suggested that 2-4 additional pseudorandom levels be used to avoid this negative effect. This is achieved by adding a pseudorandom noise sequence to the image signal which is then subtracted from the restored signal.

In communications channels with limited energy potential the best type of manipulation is DOFM. DOFM allows the communications channel band to be cut in half for the same freedom from interference as with OFM. The optimum channel band is equal to $\Pi = 1.5/\tau_0 = 6$ mHz where τ_0 is the duration of the sample ($\tau_0 = 2/B$) and B is the rate of transmission. The energy loss with respect to the potential freedom from interference, due to the less than optimum filtering and intersymbol interferences is equal to 1.5 dB. However, the band for the communications circuit can be narrowed to 4.4 mHz which increases the energy loss to 2.5 dB.

The potential freedom from interference for reception of DOFM signals (delta relative frequency modulation) is determined by the expression $P_{\text{ош}} \approx 1-\Phi(q)$, in which $q^2 = P_c \tau_0/N_0 = 2P/N_0B$. P_c is the signal's power, N_0 is the spectral density of the noise, and $\Phi(x)$ is the probability integral.

A signal/noise ratio at the input to the detector $P_c/P_{\text{ш}} = 28$ (taking into account the nonideality of the filtration and synchronization and the intersymbol interferences) is required to prove a certain probability of error for the reception of DOFM signals ($P_{\text{ош}} = 10^{-4}$), i.e., the ratio of the signal's power to the spectral density of the noise which is needed at the input to the receiver should be equal to $P_c/N_0 = 1.3 \cdot 10^8$ sec^{-1} (for a 4.4 mHz band).

A comparison of the freedom from interference for the transmission of a videotelephone image with IKM-DOFM and an

FM analog shows that for a channel band $\Pi = 4.4$ mHz and a modulation index $B = \Pi/2F_B - 1 = 1.2$, the image of analogous quality (35 dB) is achieved for a $P_c/N_0 \approx 2.0 \cdot 10^8$ $\sec^{-1}$, i.e., for FM the transmitter's power must be 1.5 times greater.

The power for the FM signal can be lowered 4.7 times if the index of modulation is increased to 2.6 but in this case the band of the communications system is increased to 7.2 mHz.

The band of the transmitted signal must be narrowed to the maximum extent in channels with a high energy potential (cable and radio relay communications lines). In this case the best method for discrete modulation is an 8-level DIKM combined with a multilevel (8-level) single band OFM (relative phase modulation). The communications channel band, in this case, must be (1.1 - 1.2) mHz, i.e., the same as that for single band AM.

The freedom from interference for the reception of a multilevel, one band, OFM signal is much less than that for DOFM.

Calculations show that the value of the ratio is $P_c/N_0 = 9.75 \cdot 10^8$ $\sec^{-1}$, for the required value $P_{\text{ош}} = 10^{-6}$ for a limited medium power and $P_c/N_0 = 3.76 \cdot 10^9$ $\sec^{-1}$ for a limited peak power of the transmitter. The required value of the ratio is $P_c/N_0 = 5.3 \cdot 10^9$ $\sec^{-1}$ for the transmission of a videotelephone image by means of single band AM (taking into account the synchrosignal and the limited peak power of the transmitter).

Thus, the interference-free transmission of image signals with discrete signals is no worse than, and in individual cases

it is better than, the freedom from interference for the transmission by analog types of modulation.

In all of the calculations whose results were given above it was assumed that the excessive signal for the image is not avoided. If the excesses in the signal were avoided the communications channel band can be reduced by several times.

All of the methods for compressing the frequency spectrum by reducing the redundancy of the videosignal image can be divided into two main groups:

1) those which provide for the artificial decrease in the precision of perceiving the image and which take into account the properties of the signal's source and its receivers.

2) those for which the precision of perceiving each element of the image is retained but only those elements are transmitted along the communications channel which differ markedly from the preceding elements.

The method of adaptive quantization of the videosignal, which takes into account the properties of human vision which allows a coarser quantization of the high frequency components in the videosignal's spectrum to be used, which is very promising, should be included in the first group. The discharging of the code for the analog-digital converter can be changed automatically as a function of the tracking frequency of the new elements of the images which are fixed by an intermediate memory. Calculation shows that for the majority of the videotelephone images the rage of transmitting information can be lowered by no less than 3-5 times using adaptive quantization.

The method of differential coding of the videosignal, which was mentioned above and which uses its interelement, interline, and interframe correlation, is in the second group. If the DIKM system with interelement correlation lowers the rate of transmission by 2-3 bits for each reading as compared with ordinary IKM [3], then systems which use memory on the number of elements in a line can reduce the rate of transmitting information by 4.5 times and systems with a memory capacity in a single frame (of the order of 100,000 elements, T_{frame} - 1/25 sec) can reduce it 20-30 times. It should be noted that reducing the superfluousness increases the efficiency of the communications channel however it lowers its freedom from interference. Calculations and experiments have shown that for communications channels with a low energy potential it is expedient to use adaptive quantization with $P_{oш}$ of the order of 10^{-5} - 10^{-6} which gives a coefficient of compression information by 3-5 times.

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Multiposition Pick Up Systems

V.A. Kisel' and I.P. Panfilov

The synthesis of multiposition coherent systems with reception in a coordinated filter and with a correlation method of reception which automatically minimize the mean square error in reception for variations in the channel parameters for a channel with an arbitrary character of the linear distortions and additive noise is discussed. The property of convexity was proven for the mean square error.

Recently much progress has been made in the theory for synthesizing coherent pick-up receivers for discrete signals [1-3]. However, the synthesis of multiposition pick-up coherent systems, which are used for reception in coordinated filters or in the correlation method of reception, have not been discussed in the literature. In this article the synthesis of such systems is discussed with probing (sond) pulses which automatically minimize the square norm of the error matrix at the outputs of the system during the transmission of signals in channels with changing parameters.

Pick-up systems with coordinated filters. The scheme for a multiposition system with coordinated filters is shown in Fig. 1 [4] where $\varphi_k(t)$ $(k=1, 2, \ldots, m)$ is a group of given working signals with the spectra $\Phi_k(\omega)$; $K_\tau(\omega)$ is a complex transmission coefficient for the parametric channel with linear distortions. RU is the resolving device, $\psi_j(\omega)$ $(j=1, 2, \ldots m)$ are complex transmission coefficients for the coordinated filters which are realized in terms of the polynomial structure

$$\psi_j(\omega) = \sum_{\nu=1}^{q} a_\nu^{(j)} \gamma_\nu^{(j)}(\omega) \quad (j = 1, 2, \ldots m),$$

where $\gamma_\nu^{(j)}(\omega)$ $(\nu=1, 2, \ldots, q;\ j=1, 2, \ldots, m)$ - given base functions, $a_\nu^{(j)}$ are controllable parameters by means of which the required transmission coefficient is attained for the coordinated filters, and $N_\tau(t)$ is an additive noise with the spectrum $N_\tau(\omega)$.

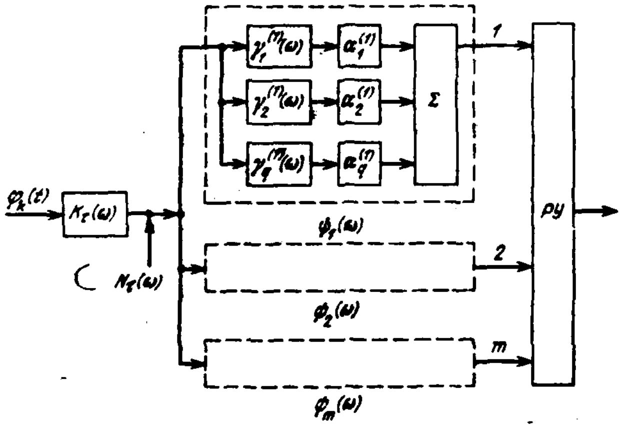

Fig. 1

If a signal $\varphi_k(t)$ is fed to the input of the channel we get

$$p'_{kj} = \sum_{\nu=1}^{q} a_\nu^{(j)} P_{kj}^{(\nu)},$$

at the j-th output of the system at the moment of recording t_0, in which

$$P_{kj}^{(\nu)} = \frac{1}{2\pi} \int_E \gamma_\nu^{(j)}(\omega) [K_\tau(\omega)\varphi_k(\omega) + N_\tau(\omega)]\, e^{i\omega t_0}\, d\omega, \qquad (1)$$

where E is the frequency multiple which encompasses the spread of the signal's spectrum at the system's output.

The system's quality will be evaluated from the square of the norm for the error matrix at the system's outputs

$$\| \eta \| = \overline{\| [\rho]' - [\rho] \|} =$$
$$= \sum_{kj=1}^{m} (\rho'_{kj} - \rho_{kj})^2 = \sum_{kj=1}^{m} \left(\sum_{\nu=1}^{q} a_\nu^{(j)} P_{kj} - \rho_{kj} \right)^2, \qquad (2)$$

where $[\rho]'$ is a matrix made up of the elements $\rho'_{kj} (k, j = 1, 2, \ldots, m)$; $[\rho]$ is a given (standard) matrix made up of the elements ρ_{kj} which must be attained at the system's outputs. Here the ' indicates averaging over the entire realizations of noise.

It is not difficult to see, by taking (1) into account, that for fixed $\gamma_\nu^{(j)}(\omega)$, $\varphi_k(\omega)$ and $K_\tau(\omega)$ the value of $\|\eta\|$ is a multidimensional function of the variables $a_\nu^{(j)}$ $(\nu = 1, 2, \ldots, q;\ j = 1, 2, \ldots, m)$, which is convex at the bottom and which does not contain any unresolved "dips."

We will assume that the coordinated filters are optimum if the value of $\|\eta\|$ is the smallest possible. The minimum for $\|\eta\|$ is found from the condition

$$\frac{\partial \| \eta \|}{\partial a_\nu^{(j)}} = 0 \qquad (\nu = 1, 2, \ldots, q;\ j = 1, 2, \ldots, m). \qquad (3)$$

These relationships are the starting equations for calculating the optimum values of the coefficients $a_\nu^{(j)}$.

The main question in the theory of synthesizing pick-up systems is the selection of an algorithm which will guarantee that we will get the global minimum for a given, useful, function. Formally, such an algorithm can be any mathematical algorithm for seeking the minimum for functions with many variables. However, proceeding from the requirements that

the technical realizaton must be simple, two of these algorithms will be discussed here: an algorithm for consecutive optimization (method of cross sections) and an algorithm for the most rapid decrease (gradient method) [5].

The method of cross sections consists in the consecutive minimizing of the quantity $\|\eta\|$ by alternately controlling the coefficients $\alpha_\nu^{(j)}$ $(\nu=1, 2, \ldots, q; j=1, 2, \ldots, m)$. To realize this method at the outputs j = 1, 2, ..., m we must include a device which indicates the value of $\|\eta\|$, for example, the circuit in Fig. 2.

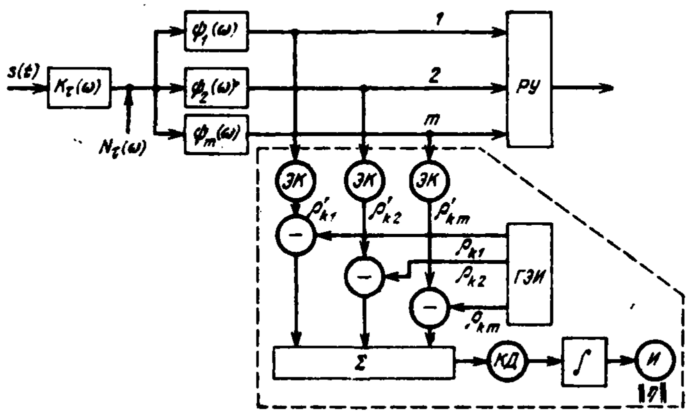

Fig. 2

A probing (sond) signal is fed into the input of the channel $K_\tau(\omega)$ in the form of a periodic sequence of operating signals $\varphi_1(t), \varphi_2(t), \ldots, \varphi_m(t)$, which are shifted by an interval T from one another (T is chosen as greater than the duration of each of the indicated signals and large enough so that the statistical properties of the noise $N_\tau(t)$ will appear

after a period of time T. Electronic switches (ES) carry out the simultaneous selection of the signals at each of the outputs 1, 2,..., m at moments of time t_0, t_0+T, t_0+2T, $t_0 + (m-1)T$, and as a result, we have a group of pulses with the amplitudes $\rho'_{1j}, \rho'_{2j}, \rho'_{3j}, \ldots, \rho'_{mj}$ $(j = 1, 2, \ldots, m)$.

Pulses with amplitudes of $\rho_{1j}, \rho_{2j}, \rho_{3j}, \ldots, \rho_{mj}$ $(j=1, 2, \ldots, m)$. are supplied by the generator of standard pulses (GSP) which are synchronized with these pulses at the deducting device. The indicator (I), which is connected to the summation device's output through the quadratic detector QD and the integrator indicates the value of $\|\eta\|$. By varying the coefficients $\alpha_{\nu}^{(j)}$, in sequence, we can find the minimum reading on I, which, because of the convexity and the absence of "dips" is the unconditional minimization for $\|\eta\|$.

For $\|\eta\|$ (2) the following form of recording is valid

$$\|\eta\| = \sum_{j=1}^{m} \eta_j,$$

where

$$\eta_j = \sum_{k=1}^{m} \overline{(\rho'_{kj} - \rho_{kj})^2} = \sum_{k=1}^{m} \overline{\left(\sum_{\nu=1}^{q} \alpha_{\nu}^{(j)} P_{kj}^{(\nu)} - \rho_{kj}\right)^2}.$$

It is obvious that for given values of $\gamma_{\nu}^{(j)}(\omega)$, $\varphi_k(\omega)$ and $K_{\nu}(\omega)$ η_j are mutually independent functions of the variables $\alpha_{\nu}^{(j)}$. Therefore, the channels 1, 2,...,m can be tuned independently and a QD, a generator, and an indicator which gives the value of η_j are connected at the output of each channel, The latter indicating the minimum value of η_j by which the given channel is tuned.

Minimizing $\|\eta\|$ by the gradient method consists of a simultaneous changing of all of the coefficients $\alpha_{\nu}^{(j)}$ $(\nu = 1, 2, \ldots, q$; and j = 1, 2,...,m) respectively by values which are proportional to the components of the gradient for the function $\|\eta\|$ and opposite to them in sign.

The gradient for $\|\eta\|$ is determined by the expression

$$\operatorname{grad} \|\eta\| = \sum_{j=1}^{q} \sum_{\nu=1}^{m} \lambda_{\nu}^{(j)} \vec{\alpha}_{\nu}^{(j)},$$

where $\lambda_{\nu}^{(j)}$ are the gradient's components and $\vec{\alpha}_{\nu}^{(j)}$ - are unit vectors with respect to the variable $\alpha_{\nu}^{(j)}$,

$$\lambda_{\nu}^{(j)} = \frac{\partial \|\eta\|}{\partial \alpha_{\nu}^{(j)}} = 2 \sum_{k=1}^{m} \overline{(\rho_{kj}' - \rho_{kj}) P_{kj}^{(\nu)}}.$$

The scheme for the system with pickup using the algorithm for the fastest decrease is shown in Fig. 3. Only the j-th channel is shown here since all of the channels are analogous in their construction and are tuned independently.

As in the scheme in Fig. 2, a periodic signal s(t) is fed into the input to the channel $K_{\tau}(\omega)$. At the outputs to the ES of the j-th channel pulses are obtained with amplitudes of $P_{kj}^{(\nu)}$ and ρ_{kj}', and at the outputs to the remultiplier pulses are obtained with amplitudes of $(\rho_{kj}' - \rho_{kj}) P_{kj}^{(\nu)}$ $(\nu = 1, 2, \ldots, q;\ k = 1, 2, \ldots, m)$. These pulses are added up and averaged in the accumulating devices Y and controlling signals are obtained at their outputs with amplitudes which are proportional to $\lambda_{\nu}^{(j)} = 2\sum_{k=1}^{m} \overline{(\rho_{kj}' - \rho_{kj}) P_{kj}^{(\nu)}}$.

The controlling signals change the transmission coefficients of the regulators $\alpha_{\nu}^{(j)}$ by a value $a\lambda_{\nu}^{(j)}$, where a is a proportionality coefficient. As a result the value of $\|\eta\|$, is minimized by the method of the fastest decrease. The described minimization is unconditional, i.e., we are describing the global minimum for $\|\eta\|$ since $\|\eta\|$ is a convex function which does not have "dips."

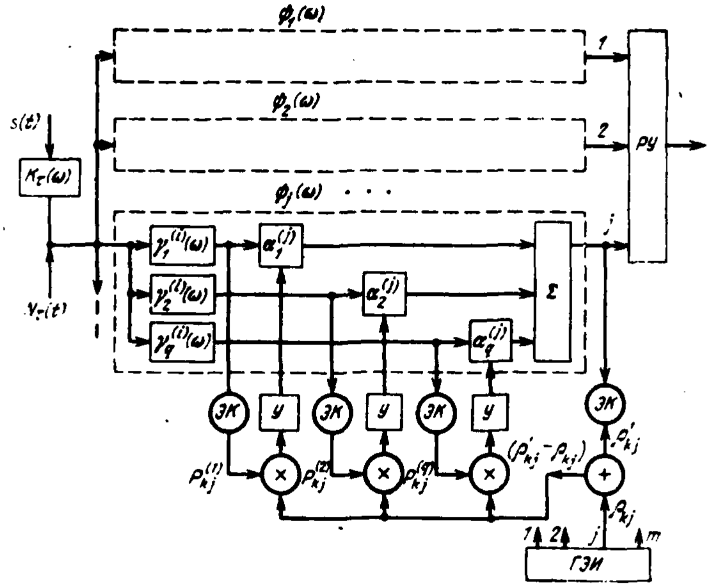

Fig. 3

Pickup systems with correlation reception. A multiposition system with correlation reception is shown in Fig. 4 [4] where $g_\tau(t)$ is the pulsed reaction for the parametric channel with transmission coefficients $K_\tau(\omega)$; $\psi_j(t)$ $(j=1, 2, \ldots, m)$ are the supporting signals produced by the supporting generator Γ_j $(j=1, 2, \ldots, m)$. Each of the supporting generators is fashioned in the form of a group of generators of the base signals $\gamma_\nu^{(j)}(t)$ $(\nu=1, 2, \ldots, q)$, whose outputs are switched to the appropriate summators through the controlling attenuators $\alpha_\nu^{(j)}$ Therefore, the supporting signals can be represented in the form

$$\psi_j(t)=\sum_{\nu=1}^{q} \alpha_\nu^{(j)} \gamma_\nu^{(j)}(t) \quad (j=1, 2, \ldots, m).$$

A change in the coefficients $\alpha_\nu^{(j)}$ gives the shape of the signal which is needed $\psi_j(t)$. .

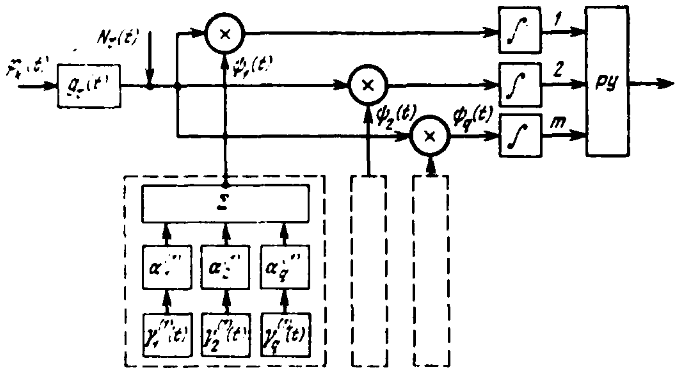

Fig. 4

We get the following for the transmission of the k-th signal $\varphi_k(t)$ at the outputs for the integrators

$$\rho'_{kj} = \sum_{\nu=1}^{q} \alpha_\nu^{(j)} Q_{kj}^{(\nu)};$$

$$Q_{kj}^{(\nu)} = \int_{E_t} [f_k(t) + N_\tau(t)]\, \gamma_\nu^{(j)}(t)\, dt;$$

$$f_k(t) = \int_{-\infty}^{t} \varphi_k(\theta)\, g_\tau(t-\theta)\, d\theta,$$

where E_t is the multiple by which the integration is made (specifically E_t - the multiplicity of points on the segment $[0, t_0]$; t_0 is the moment of recording). The system's quality is evaluated in terms of the averaged square of the norm for the error matrix at the outputs of the integrators

$$\|\eta\| \overline{\|[\rho]' - [\rho]\|} = \sum_{kj=1}^{m} \left(\sum_{\nu=1}^{q} \alpha_\nu^{(j)} Q_{kj}^{(\nu)} - \rho_{kj}^2 \right). \quad (4)$$

where $[\rho]'$ is the matrix of the parameters being used, ρ'_{kj}; $[\rho]$ is the standard matrix. The averaging is made over all of the noise realizations.

Equation (4) does not differ in form from (2), i.e., for given values of $\varphi_k(t)$ $(k=1, 2, \ldots, m)$ $g_\nu(t)$ and $\gamma_\nu^{(j)}(t)$ $(j=1, 2, \ldots, m;\ \nu=1, 2, \ldots, q)$ $\|\eta\|$ is a function of the variables $\alpha_\nu^{(j)}$ $(\nu=1, 2, \ldots, q;\ j=1, 2, \ldots, m)$, which is convex at the bottom and which does not have any unresolved "dips." Therefore, the methods of cross sections and gradients will be used to minimize $\|\eta\|$.

In systems with correlation reception the method of cross sections does not differ from the same method in systems with coordinated filters and it is carried out by connecting a device (Fig. 4), which determines the value of $\|\eta\|$, and which has a construction analogous to the device in Fig. 2, to the outputs of the integrators.

The gradient system, which makes use of the algorithm for the fastest decrease is shown in Fig. 5, in which, for simplicity, only one, the j-th, channel is shown. The coefficients $\alpha_\nu^{(j)}$ $(\nu=1, 2, \ldots, q)$ for this system change a value $a\lambda_\nu^{(j)}$, for which

$$\lambda_\nu^{(j)} = \frac{\partial \|\eta\|}{\partial \alpha_\nu^{(j)}} = 2 \sum_{k=1}^{m} \overline{(\rho_{kj}' - \rho_{kj})\, Q_{kj}^{(\nu)}}$$

(a is a proportionality coefficient. All of the system's channels are identical in their construction and are tuned, independently to the signal s(t).

We get pulses at the outputs for the generators with the amplitudes $Q_{kj}^{(\nu)}$ and ρ_{kj}', and pulses with amplitudes $(\rho_{kj}' - \rho_{kj})\, Q_{kj}^{(\nu)}$, are obtained at the outputs to the re-multiplier which are summed and averaged in the accumulators Y being controlled by the regulators $\dot{\alpha}_\nu^{(j)}$ $(\nu=1, 2, \ldots, q)$.

The systems which were described are adapted by means of the test signal. This is either done in the gaps between communications sessions or directly in the process of transmitting information by mixing in periodic test signals with the working signal in transmission and separating them by the method of accumulating this signal in reception.

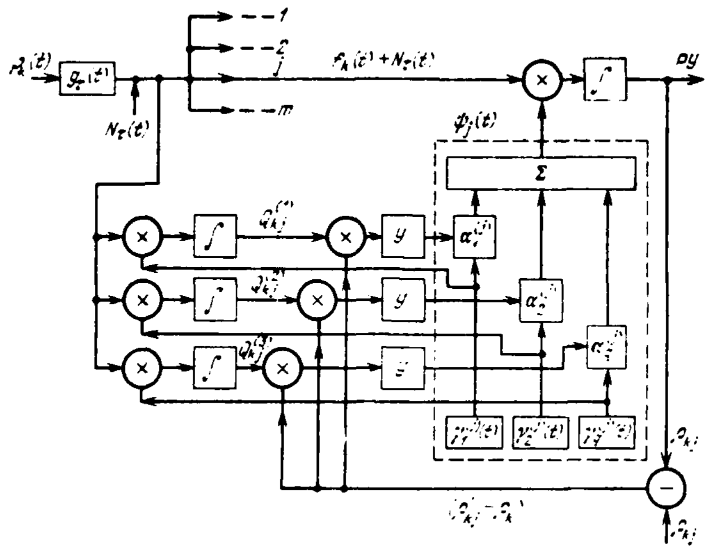

Fig. 5

The advantage of these systems is that they guarantee achieving a minimum for $\|\eta\|$ in the presence of additive noises and for arbitrary linear distortions in the channel.

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Optimization of the Base Functions for Multibeam Channels with Linear Distortions

I.P. Panfilov

Integral equations were found for minimizing the mean square error in multibeam, multiposition systems for the combined action of linear distortions and additive noise in the channel.

Studies [1-3] were devoted to an analysis of the freedom from interference for communication systems with multibeam propagation of the signals in the channel. However, up to now, the question of the synthesis of the base functions, by which we mean the working or supporting signals and the transmission coefficients of the coordinated filters, which would allow the probability of error to be minimized in multiposition systems for the combined action of multibeam propagation, linear distortions, and additive noise remains unsolved. In this article, this problem is solved on the basis of the classical variation calculations using coherent systems as the example.

Let us consider a system with a receiver in the form of coordinated filters (Fig. 1) [1]. The following designations are used: $\varphi_k(\omega)$ $(k=1, 2, \ldots, m)$ is a group of working signals, $\psi_j(\omega)$ $(j=1, 2, \ldots, m)$ are complex transmission coefficients for the coordinated filters, $K_r(\omega,\tau)$ are complex transmission coefficients for the channel with an arbitrary type of linear distortions, $N(\omega)$ is the additive noise, μ_r are transmission coefficients which characterize the attenuation of the signal which travels by different paths, and Δt_r is the delay time of the r-th beam.

For the transmission of the k-th signal $\varphi_k(\omega)$, we get the following at the outputs of the filters at the moment of

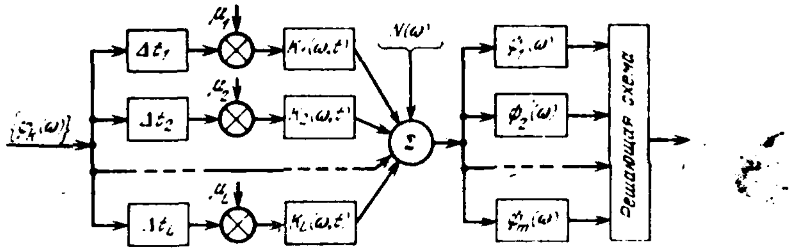

Fig. 1

reading t^ν in the presence of linear distortions in the channel and for multibeam propagation

$$\rho'_{ky}(t^\nu) = \frac{1}{2\pi}\int_E \left\{\varphi_k(\omega)\left[\sum_{z=1}^{L}\mu_r K_r(\omega,\tau)\times\right.\right.$$

$$\left.\left.\times e^{-\Delta t_r i\omega}\right] + N(\omega)\right\}\psi_j(\omega)\, e^{i\omega t^\nu}\, d\omega, \qquad (1)$$

where E is the frequency multiple by which the modulus of the spectra for the received signals differs from zero.

We will assume that the communications system provides for a minimum error if $\rho'_{kj}(t^\nu) = \rho_{kj}(t^\nu)$, where $\rho_{kj}(t^\nu)$ are given values at the points of reading. In the given channels the change in the parameters and the presence of noise means that this equation will not be fulfilled for all values of τ, Δt, μ and different realizations for the noise $N(\omega)$. We will evaluate the quality of the system from the value of the square of the norm for the error matrix

$$\|\eta\| = \overline{\|[\rho'_{kj}(t^\nu)] - [\rho_{kj}(t^\nu)]\|} =$$

$$= \sum_{k,j=1}^{m}\overline{[\rho'_{kj}(t^\nu) - \rho_{kj}(t^\nu)]} = \sum_{k,j=1}^{m}\{\overline{[\rho'_{kj}(t^\nu)]^2} -$$

$$- 2\rho_{kj}(t^\nu)\overline{\rho'_{kj}(t^\nu)} + \rho^2_{kj}(t)^\nu\} \qquad (2)$$

(here and in the following the ' indicates averaging in terms of τ and for all realizations of the noise).

Let us solve the problem: determine the signal $\varphi_k(\omega)$, which allows $\overline{\|\eta\|}$ to be minimized for given values of $\psi_j(\omega)$, $K_r(\omega, \tau)$ and Δt_r.

After the appropriate transformations taking (1) into consideration, we get

$$\overline{\|\eta\|} = \sum_{k=1}^{m} P_k, \tag{3}$$

in which

$$P_k = \int\limits_E\int\limits_E \varphi_k(\omega)\,\varphi_k(\Omega) K_k(\omega, \Omega)\, d\omega d\Omega -$$

$$-2\int\limits_E \varphi_k(\omega) B_k(\omega)\, d\omega + D,$$

$$K_k(\omega, \Omega) = \frac{1}{4\pi^2}\sum_{j=1}^{m}\left\{\left[\overline{\sum_{r=1}^{L} \mu_r K_r(\omega, \tau)\, e^{-\Delta t_r i\omega}}\right]\times\right.$$

$$\left.\times\left[\sum_{r=1}^{L} \overline{\mu_r K_r(\Omega, \tau)\, e^{-\Delta t_r i\Omega}}\right]\psi_j(\omega)\,\psi_j(\Omega)\, e^{it^{\nu}(\omega+\Omega)}\right\};$$

$$B_k(\omega) = \frac{1}{2\pi}\sum_{j=1}^{m}\left\{\overline{\sum_{r=1}^{L} \mu_r K_r(\omega, \tau)\, e^{-\Delta t_r i\omega}\,\psi_j(\omega)\, e^{i\omega t^{\nu}}}\times\right.$$

$$\left.\times\left[\rho_{kj}(t^{\nu}) - \frac{1}{2\pi}\overline{\int\limits_E N(\omega)\,\psi_j(\omega)\, e^{i\omega t^{\nu}}}\right]\right\};$$

$$D = \sum_{k, j=1}^{m}\left\{\rho_{kj}^2(t^{\nu}) + \frac{1}{2\pi}\overline{\int\limits_E N(\omega)\,\psi_j(\omega) e^{i\omega t^{\nu}}\, d\omega}\times\right.$$

$$\left.\times\left[\overline{\frac{1}{2\pi}\int\limits_E N(\omega)\,\psi_j(\omega)\, e^{i\omega t^{\nu}}\, d\omega} - 2\rho_{kj}(t^{\nu})\right]\right\}.$$

In order for a solution to exist for the equation which was found, it is necessary and sufficient that $P_k \geqslant 0$. It is not difficult to see that

$$P_k = \sum_{j=1}^{m} \overline{[\rho'_{kj}(t^{\nu}) - \rho_{kj}(t^{\nu})]^2} \geqslant 0,$$

consequently, each term in the right hand side (3) is positive. The minimum for P_k is attained for [4]

$$\int_E \varphi_k(\Omega) K(\omega, \Omega)\, d\Omega - B_k(\omega) = 0 \quad (k = 1, 2, \ldots, m), \quad (4)$$

i.e., the optimum values of $\varphi_k(\omega)$ for which $\|\eta\|$ is minimal are the solution to the linear integral first order equation. We will limit ourselves to a study of the signals $\varphi_k(\omega)$ with finite energy, i.e.

$$\int_{-\infty}^{\infty} \varphi_k^2(t)\, dt < \infty.$$

We will find the minimum for P_k for the limitation

$$\int_{-\infty}^{\infty} \varphi_k^2(t)\, dt = \frac{1}{2\pi} \int_E |\varphi_k(\omega)|^2\, d\omega = \text{const}$$
$$(k = 1, 2, \ldots, m). \qquad (5)$$

Thus, the optimum signal, taking the limitation (5) into account, is the solution of the nonhomogeneous, integral equation of the second order [4]:

$$\frac{\lambda_k}{2\pi} \varphi_k^*(\omega) + \int_E \varphi_k(\Omega) K_k(\omega, \Omega)\, d\Omega -$$
$$- B_k(\omega) = 0 \quad (k = 1, 2, \ldots, m), \qquad (6)$$

in which λ_k is the Lagrange multiple (*indicates complex conjugation).

Variation of the formulation of this problem. Let us find the optimum transmission coefficients for the coordinated filters $\psi_j(\omega)$, which minimize $\|\eta\|$ for the case in which the signals $\varphi_k(\omega)$ and the linear distortions $K_r(\omega, \tau)$ are given and they change as a function of the parameters τ and Δt_r.

By analogy with the foregoing, taking (1) into account and making the appropriate transformations in (3), the minimum P_k is reached for

$$\int_E \psi_j(\omega) F_j(\omega, \Omega)\, d\Omega - C_j(\omega) = 0, \tag{7}$$

where

$$F_j = (\omega) = \frac{1}{4\pi^2} \sum_{i=1}^{m} e^{it_i^{\nu}(\omega+\Omega)} \times$$

$$\times \left\{ \left[\sum_{r=1}^{L} \mu_r K_r(\omega, \tau) e^{-\Delta t_r i\omega} \right] \left[\sum_{r=1}^{L} \mu_r K_r(\Omega, \tau) \times \right.\right.$$

$$\left. \times e^{-\Delta t_r i\Omega} \right] \varphi_k(\omega) \varphi_k(\Omega) + 2 \sum_{r=1}^{L} \mu_r K_r(\omega, \tau) \times$$

$$\left. \times e^{-\Delta t_r i\omega} N(\Omega) \varphi_k(\omega) + N(\omega) N(\Omega) \right\};$$

$$C_j(\omega) = \frac{1}{2\pi} \sum_{k=1}^{m} \rho_{kj}(t^{\nu}) e^{i\omega t^{\nu}} \times$$

$$\times \left[\sum_{r=1}^{L} \mu_r K_r(\omega, \tau) e^{-\Delta t_r i\omega} \varphi_k(\omega) + N(\omega) \right].$$

Applying the following limitation to $\psi_j(\omega)$

$$\int_E |\psi_j(\omega)|^2 d\omega = \text{const}$$

the optimum transmission coefficient for the coordinated filter which minimizes $\|\eta\|$, taking this limitation into account, will satisfy the integral equation

$$\frac{\lambda_j}{2\pi} [\psi_j^*(\omega)] + \int_E \psi_j(\Omega) F_j(\omega, \Omega)\, d\Omega - C_j(\omega) = 0. \tag{8}$$

It should be pointed out that the equations (6) and (8) that were obtained are valid for both coherent and integral reception. If $\psi_j(\omega) = 1$, in equation (6), then we will have integral reception. If $\psi_j(\omega)^*$ in equations (6) and (8) is replaced by $\varphi_j(\omega)$, then we get coherent reception [2].

Thus, equations (6) and (8) can be used to find either the optimum working or supporting signals or the optimum transmission coefficients for the coordinated filters which minimize the error in multiposition, multibeam systems for the combined action of linear distortions and additive interferences in the channel.

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Analysis of the Energy Losses for the Action of Intersymbol and Interchannel Interferences in Multichannel Systems with Phase Manipulation

Yu. F. Korobov and A.L. Federov

The freedom from interference was studied for multichannel, asynchronous systems with phase manipulation for the simultaneous action of intersymbol and interchannel interferences and a fluctuation noise. It is assumed that the system contains transmitting and receiving filters. The losses are given as a function of the base of the communications system and the relative detuning between the channels.

As a rule frequency packing of the channels is used in satellite communications systems based on multichannel operation with "free access." In this the problem arises of making the optimum selection of the transmission bands for the channel filters which minimize the equivalent energy losses due to intersymbol interferences in the given channel and interchannel interferences for neighboring channels. The optimum band is related to the fact that the narrowing of the transmission band for the channel filters during transmission and reception reduces the effect of the interchannel interferences as well as of different types of interferences which are in the signal's spectrum but in this case the action of the intersymbol interferences increases.

We know that the best solution for the problem of separating the channels is the use of frequency-time principle (Kinepleks [3], MS-5 [2]) which, in the ideal case, means that for a minimum total band we can be rid of the intersymbol and the interchannel interferences. However, since this method requires synchronized

operation for all of the channels, then it is not applicable to the given system since in real units there are energy losses associated mainly with the incomplete use of the duration, i.e., there are losses of the signal's sample. Because of this the problem is solved of finding the optimum bands for filters in systems with asynchronous channels which correspond to a minimum for the energy losses for a coherent DFT with frequency separation of the channels and a comparison is made of these systems with systems with frequency-time separation as to the efficiency for using the signal's energy and frequency band.

Transition characteristic of a channel in a multichannel, coherent system. In order to determine the intersymbol and interchannel interferences we must have some knowledge of the response at the output of the coherent detector of a given channel to a single jump at the input of any channel filter in the transmitter. We will assume that this is a low frequency filter connected in the videosignal's circuit which has the transition characteristic h(t). Then the response at the output of the transmitter will be equal to

$$\dot{H}_{\text{пер}}(t) = h(t)\, e^{j(\omega t+\varphi)}, \qquad (1)$$

and after it passes through the receiving band filter of the given channel, according to the Duhamel integral it assumes the form

$$\dot{H}_{\text{пф}}(t) = \int_0^t g(\tau)\, \dot{H}_{\text{пер}}(t-\tau)\, d\tau = \int_0^t g(\tau)\, h(t-\tau)\, e^{-j\omega\tau}\, d\tau\, e^{j(\omega t+\varphi)}, \qquad (2)$$

where

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K(j\omega)\, e^{j\omega t}\, d\omega = \frac{1}{\pi}\int_0^{\infty} K(\omega)\cos[\omega t + \psi(\omega)]\, d\omega \qquad (3)$$

is the pulsed reaction of the band filter.

In the following it will be assumed that the band filter, because of its narrow band, allows us to approximate the transmission coefficients $K(j\omega)=\dot{K}(\omega)e^{j\psi(\omega)}$ which are symmetrical with respect to the mean frequency for the filter ω_0 by the functions

$$\begin{aligned} K(\omega_0+\Omega) &= K(\omega_0-\Omega) \\ \psi(\omega_0+\Omega) &= -\psi(\omega_0-\Omega) \end{aligned} \quad \text{при } |\Omega| \leqslant \omega_0. \tag{4}$$

Because of this the function g(t) can be expressed in terms of the pulsed reaction of the low frequency equivalent filter

$$g_э(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}K_э(j\Omega)\,e^{j\Omega t}\,d\Omega, \tag{5}$$

where $K_э(j\Omega)=K_э(\Omega)\,e^{j\psi_э(\Omega)},\ K_э(\Omega)=K(\omega_0+\Omega),$

$$\psi_э(\Omega)=\psi(\omega_0+\Omega). \tag{6}$$

By using (4) we can write (3) in the form

$$g(t)=\frac{1}{\pi}\int_{-\omega_0}^{\omega_0}K(\omega_0+\Omega)\cos[(\omega_0+\Omega)t+\psi(\omega_0+\Omega)]d\Omega=$$

$$=\frac{2}{\pi}\int_{0}^{\omega_0}K_э(\Omega)\cos[\Omega t+\psi_э(\Omega)]\,d\Omega\cos\omega_0 t.$$

Taking into account the naarrowness $K_э(\Omega)$ as compared with ω_0 equation (5) we can write

$$g(t)=\frac{2}{\pi}\int_{0}^{\infty}K_э(\Omega)\cos[\Omega t+\psi_э(\Omega)]\,d\Omega\cos\omega_0 t=$$

$$=2g_э(t)\cos\omega_0 t. \tag{7}$$

By substituting (7) in (2), we get

$$\dot{H}_{пф}(t)=\int_{0}^{t}g_э(\tau)\,h(t-\tau)\,[e^{-j(\omega-\omega_0)\tau}+$$

$$+e^{-j(\omega-\omega_0)\tau}]\,d\tau\,e^{j(\omega t+\varphi)}.$$

Since the second term in the brackets oscillates rapidly then, finally

$$\dot{H}_{n\phi}(t) = \int_0^t g_0(\tau)\, h(t-\tau)\, e^{-j\Delta\omega\tau}\, d\tau\, e^{j(\omega t+\varphi)}, \qquad (8)$$

where $\Delta\omega = \omega - \omega_0$.

If the response (8) and a supporting oscillation equal to $\cos(\omega_{on} t + \varphi_{on})$, act on the inputs of the coherent detector then at the input to the detector the response has the form

$$\dot{H}(t) = \int_0^t g_0(\tau)\, h(t-\tau)\, e^{-j\Delta\omega\tau}\, d\tau\, e^{j(\delta\omega t+\gamma)}, \qquad (9)$$

where $\delta\omega = \omega - \omega_{on}$ и $\gamma = \varphi - \varphi_{on}$.

By separating out the real part of equation (9), we will write the voltage at the output of the coherent detector, which corresponds to the action of an interference $\omega = \omega_n$ on the frequency as

$$H_n(t, \Delta\omega_n) = \int_0^t g_0(\tau)\, h(t-\tau) \cos(\delta\omega_n t - \Delta\omega_n \tau + \gamma_n)\, d\tau. \qquad (10)$$

For precise tuning, when $\omega_{on} = \omega_0$ и $\delta\omega_n = \Delta\omega_n$

$$H_{n0}(t) = \int_0^t g_0(\tau)\, h(t-\tau) \cos[\Delta\omega_n (t-\tau) + \gamma_n]\, d\tau. \qquad (11)$$

here $\delta\omega_n = \omega_n - \omega_{on}$, $\Delta\omega_n = \omega_n - \omega_0$ $\gamma_n = \varphi_n - \varphi_{on}$.

We can determine the intersymbol and interchannel interferences by using the characteristics of the coherent filters and the given formulas. For this we will discuss linear systems whose transition characteristic has the form

$$h(t_n) = 1 + \sum_{m=1}^{M} a_m e^{p_m t_n}, \qquad (12)$$

as the low frequency filters. Here $t_н = 2\pi F_н t$ is the normalized time, F_H is the transmission band of the FNCh; p_m are the roots of the corresponding transmission coefficient in the operative form [1, 4]

$$K(p) = [B(p)]^{-1} = \left[\prod_{m=1}^{M} (p - p_m)\right]^{-1}, \tag{13}$$

$$a_m = [p_m B'(p_m)]^{-1} = \left[p_m \prod_{\substack{l=1 \\ l \neq m}}^{M} (p_m - p_l)\right]^{-1}. \tag{14}$$

The transition characteristic for the low frequency equivalent band filter can be written in an analogous way

$$h_э(t_э) = 1 + \sum_{n=1}^{N} b_n e^{p_n t_э}, \tag{15}$$

where $t_э = 2\pi F_э t$ is the normalized time, $2F_э$ is the transmission band of the PF (band filter), and p_n is the root $K_э(p) = [B_э(p)]^{-1}$ and

$$b_n = [p_n B'(p_n)]^{-1} = \left[p_n \prod_{\substack{l=1 \\ l \neq n}}^{N} (p_n - p_l)\right]^{-1}. \tag{16}$$

From this the pulsed reaction is

$$g_э(t_э) = h'(t) = \sum_{n=1}^{N} p_n b_n e^{p_n t_э}. \tag{17}$$

By substituting (12) and (17) in (9) we get

$$\dot{H}(t_э) = \int_0^{t_э} h'(t_э) h(t_э - \tau) e^{-j\Delta\omega_э \tau} d\tau \, e^{j(\delta\omega_э t_э + \gamma)} =$$

$$= \int_0^{t_э} \left[\sum_{n=1}^{N} p_n b_n e^{(p_n - j\Delta\omega_э)\tau} + \sum_{m=1}^{M} \sum_{n=1}^{N} a_m p_n b_n \times\right.$$

$$\left. e^{(p_n - g p_m - j\Delta\omega_э)\tau} \right] d\tau \, e^{j(\delta\omega_э t_э + \gamma) + g p_m t_э}, \tag{18}$$

where $\Delta\omega_э = \Delta\omega / 2\pi F_э$, $\delta\omega_э = \delta\omega / 2\pi F_э$, $g = 2\pi F_н / 2\pi F_э =$

$= F_{\text{н}} F_{\text{э}}$ and $a_{m\text{э}}$ is determined by equation (14) by substituting gp_m in place of p_m.

After integrating

$$\dot{H}(t_{\text{э}}) = \Big\{ \sum_{n=1}^{N} \frac{p_n b_n}{p_n - j\Delta\omega_{\text{э}}} [e^{(p_n - j\Delta\omega_{\text{э}})t_{\text{э}}} - 1] +$$

$$+ \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{a_{m\text{э}}\, p_n\, b_n}{p_n - gp_m - j\Delta\omega_{\text{э}}} [e^{(p_n - j\Delta\omega_{\text{э}})t_{\text{э}}} -$$

$$- e^{gp_m t_{\text{э}}}]\Big\}\, e^{j(\delta\omega_{\text{э}} t_{\text{э}} + \gamma)}. \qquad (19)$$

For a signal $(\delta\omega_{\text{э}} = 0)$ for precise tuning $(\Delta\omega_{\text{э}} = \gamma = \;= 0)$, equation (19) assumes the form

$$\dot{H}_{co}(t_{\text{э}}) = 1 + \sum_{n=1}^{N} b_n e^{p_n t_{\text{э}}} +$$

$$+ \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{a_{m\text{э}}\, p_n b_n}{p_n - gp_m} (e^{p_n t_{\text{э}}} - e^{gp_m t_{\text{э}}}). \qquad (20)$$

Here the fact that $h_{\text{э}}(0) = 0$ has been taken into account and, consequently, on the basis of (12)

$$\sum_{n=1}^{N} b_n = -1. \qquad (21)$$

Let us consider the case in which the FNCh coincides with a low frequency equivalent of the band filter, i.e., $h(t) = h_{\text{э}}(t)$. Then $g = 1,\ a_{m\text{э}} = b_m$ and equation 20 is converted into the form

$$\dot{H}_{co}(t_s) = 1 + \sum_{n=1}^{N} b_n e^{p_n t_s} +$$

$$+ \sum_{m=1}^{N} \sum_{n=1}^{N} \frac{b_m p_n b_n}{p_n - p_m} (e^{p_n t_s} - e^{p_m t_s}), \qquad (22)$$

in which $p_m = p_n$ for $m = n$. Assuming that

$$\lim_{p_m \to p_n} \frac{e^{p_n t_s} - e^{p_m t_s}}{p_n - p_m} = t_s e^{p_n t_s},$$

we can write

$$\dot{H}_{co}(t_s) = 1 + \sum_{n=1}^{N} b_n (1 + p_n b_n t_s) e^{p_n t_s} +$$

$$+ \sum_{\substack{m=1 \\ m \neq n}}^{N} \sum_{n=1}^{N} \frac{b_m p_n b_n}{p_n - p_m} (e^{p_n t_s} - e^{p_m t_s}). \qquad (23)$$

Since the bands p_m and p_n belong to the same multiple, then the double sum can be represented in the form

$$\sum_{\substack{m=1 \\ m \neq n}}^{N} \sum_{n=1}^{N} b_n b_m \frac{p_n + p_m}{p_n - p_m} e^{p_n t_s}. \qquad (24)$$

Then, by substituting (24) in (23) we finally get

$$\dot{H}_{co}(t_s) = 1 + \sum_{n=1}^{N} b_n \Bigg(1 + p_n b_n t_s +$$

$$+ \sum_{\substack{m=1 \\ m \neq n}}^{N} b_m \frac{p_n + p_m}{p_n - p_m} \Bigg) e^{p_n t_s}. \qquad (25)$$

The response of the interference for $\omega_{on} = \omega_0$ (i.e., when $\delta\omega_s = \delta\omega_{sn} = \delta\omega_n$: $2\pi F_s = \Delta\omega_n / 2\pi F_s' = \Delta\omega_{sn}$) according to (19) is equal to

$$\dot{H}_{no}(t_s, \Delta\omega_{sn}) = \Bigg\{ \sum_{n=1}^{N} \frac{p_n b_n}{p_n - j\Delta\omega_{sn}} (e^{p_n t_s} - e^{j\Delta\omega_{sn} t_s}) +$$

$$+ \sum_{\substack{m=1 \\ m \neq n}}^{N} \sum_{n=1}^{N} \frac{b_m p_n b_n}{p_n - p_m - j\Delta\omega_{sn}} \left[e^{p_n t_s} - e^{(p_m + j\Delta\omega_{sn}) t_s} \right] \Bigg\} e^{j\gamma}. \qquad (26)$$

The interference-free, coherent reception of DFT signals in the presence of intersymbol, interchannel, and fluctuation interferences. The probability of errors for each of two orthogonal signals in a coherent DFT system for single reading is determined by the expression

$$P_{\text{ош}}(U) = 0{,}5\,[1 - \Phi(U/\sigma_{\text{ш}})], \qquad (27)$$

where

$$\Phi(x) = \frac{2}{\sqrt{2\pi}} \int_0^x e^{-t^2/2}\,dt, \cdot U = U_{\text{c}}(t_0) + U_{\text{мс}}(t_0) + U_{\text{мк}}(t_0)$$

is the sum of the voltages for the signal $U_C(t_0)$, the intersymbol $U_{MC}(t_0)$ and interchannel $U_{MK}(t_0)$ interferences at the moment of reading t_0, $\sigma^2_{\text{ш}} = N_0 F_{\text{ш}}$ is the dispersion for the noise, and N_0 is the energy spectrum of the noise at the input to the PF and

$$F_{\text{ш}} = \frac{1}{\pi} \int_0^\infty K_\text{э}^2(\omega)\,d\omega$$

is the noise band in the PF. The of the received sample is determined as the difference

$$U_{\text{c}}(t_0) = U_m\,[\dot{H}_{\text{c}0}(t_0) - \dot{H}_{\text{c}0}(t_0 - T)], \qquad (28)$$

where U_m is the voltage of the nondistorted sample. The voltage of the intersymbol interference is the sum

$$U_{\text{мс}}(t_0) = \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \xi_k\, U_{\text{c}}(t_0 - kT). \qquad (29)$$

Here $\xi_k = \pm 1$ is a random value which takes into account the polarity of the preceding and following samples which, in the following, will be assumed to be equally probable and independent. The voltage for the interchannel interferences is found as

$$U_{\text{мк}}(t_0) = \sum_{l=1}^{L} U_{\text{мк}}(t_0, \Delta\omega_l), \tag{30}$$

which by analogy with (28) and (29)

$$U_{\text{мк}}(t_0, \Delta\omega_l) = \sum_{k=-\infty}^{\infty} \xi_{kl} U_{\text{м}} \{\dot{H}_{\text{по}}[t_0 - kT, \Delta\omega_l] - \\ - \dot{H}_{\text{по}}[t_0 (k+1) T, \Delta\omega_l]\}$$

and $\Delta\omega_l = \omega_{\text{п}l} - \omega_0$ is the detuning of the l-th channel with respect to the given channel in a system containing L + 1 channels.

Assuming that the interchannel interference is small, we will expand $P_{\text{ош}}(U)$ in a Taylor series in powers of $U_{\text{MK}}(t_0)$:

$$P_{\text{ош}}(U) = \sum_{q=0}^{\infty} \frac{1}{q!} U_{\text{мк}}^{q}(t_0) P_{\text{ош}}^{(q)}(U_{\text{c}} + U_{\text{мс}}). \tag{31}$$

The probability $P_{\text{ош}}$ must be averaged with respect to the values of the interchannel interference. Since the system is asynchronous and the interference can be considered as an energy-favorable process, then we will average $P_{\text{ош}}(U)$ with respect to time. The function $U_{\text{MK}}(t)$ does not contain a constant component. Therefore, in equation (31) after averaging, only the even powers are retained. In this case, we will limit ourselves to the square term and we get

$$\overline{P_{\text{ош}}(U)} = P_{\text{ош}}(U_{\text{c}} + U_{\text{мс}}) + \frac{1}{2} \sigma_{\text{мк}}^2 P_{\text{ош}}''(U_{\text{c}} + U_{\text{мс}}). \tag{32}$$

The quantity $\sigma_{\text{мк}}^2 - \overline{U_{\text{мк}}^2}$ can also be calculated by taking into account two orthogonal signals in each channel

$$\sigma_{\text{мк}}^2 = 2 \sum_{l=1}^{L} \frac{1}{2\pi} \int_0^{\infty} K_{\text{пф}}^2(\omega) K_{\text{н}}^2(\omega - \omega_{\text{п}l}) G(\omega - \omega_{\text{п}l}) d\omega,$$

where $K_{пф}(\omega)$ и $K_н(\omega)$ are the moduli of the transmission coefficients for PF and FNCh and

$$G(\omega - \omega_{п\,l}) = \frac{T}{2}\left[\frac{U_m \sin(\omega - \omega_{п\,l})\,T/2}{(\omega - \omega_{п\,l})\,T/2}\right]^2 \qquad (33)$$

is the one-sided energy spectrum for the PM interference with equally probable and independent samples of amplitude U_m and the carrier frequency $\omega_{пl}$.

By using the transmission coefficient for the low frequency equivalent of the PF $K_э(\Omega_l) = K(\omega_0 + \Omega_l)$, and substituting the new variable $\Omega = \omega - \omega$ and by taking into account that the integrand functions are positive and the narrow band nature of the filters, we can finally write:

$$\sigma^2_{мк} = \frac{1}{\pi}\sum_{l=1}^{L}\int_0^\infty [K_э^2(\Omega - \Delta\omega_l) + K_э^2(\Omega + \Delta\omega_l)] \times \times K_н^2(\Omega)\, G(\Omega)\, d\Omega. \qquad (34)$$

The final expression for the probability of error is found after additional averaging over the realizations of the intersymbol interference

$$\overline{\overline{P_{ош}}} = \sum_{i=1}^{\infty} P_i\,\overline{P_{ош}(U_i)} = \sum_{i=1}^{\infty} P_i\left[P_{ош}(U_c + U_{мс\,i}) + + \frac{1}{2}\sigma^2_{мк} P''_{ош}(U_c + U_{мс\,i})\right], \qquad (35)$$

where P_i is the probability of the i-th combination of random values of ξ_k in (29).

Now we can determine the equivalent energy losses due to intersymbol and interchannel interferences which occur in the given system as compared with an ideal V.A. Kotel'nikov receiver. The coefficient of the energy losses Θ is found from the equation

$$\overline{P_{ош}} = P_{ош} = 0{,}5\,[1 - \Phi(\sqrt{2E_1/N_0\theta})]. \qquad (36)$$

where $E_1 = U_m^2 T/2$ is the energy for each of the orthogonal DFT signals and N_0 is the energy spectrum of the noise at the input to the band filter.

On the basis of the formulas which were found an analysis was made of a multifrequency DFT system for the case in which Batterworth filters are used in it (in the transmission of four-component FNCh with a band F_H = F/2 and for receiving four-component PF with a band F). The calculations were made for the case in which the signal's power at the output of the FNCh is fixed.

The dependence of Θ on the size of the base of the system B = FT is shown in Fig. 1 for a number of values of the relative detuning between neighboring channels, with respect to frequency $\Delta f_1/T$ and the dependence of the minimum values of Θ_{min} and the optimum values of the base B_{opt} which correspond to them are shown in Fig. 2 for $P_{ош} = 10^{-4}$-10^{-5}, determined by equation

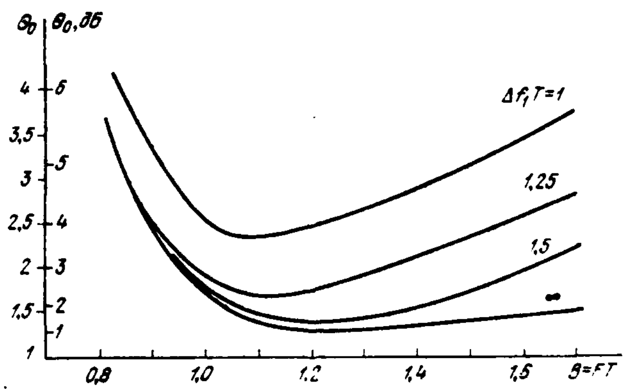

Fig. 1

(36) for $\Theta = 1$ (curve for $\Delta f_1 T = \infty$ corresponds to a one channel system, i.e., the absence of interchannel interference).

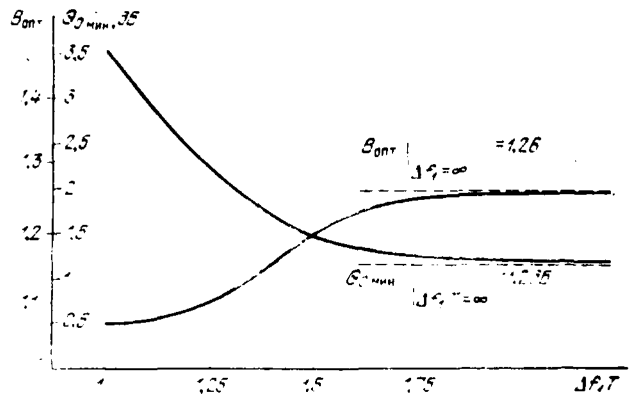

Fig. 2

We can see from Fig. 1 that the minimum loss Θ is rather dull. Therefore, the selection of the base B_{opt} and the optimum band is not very critical.

In the MS-5 system [2], the value $\Delta f_1 T = 1.18 - 1.42$ and, consequently, the losses due to the incomplete use of the duration, i.e., the energy of the sample, is 0.7-1.5 dB. In this system, according to Fig. 2, for the same values of $\Delta f_1 T$ the losses are equal to 2.7 - 1.7 dB, i.e., worse by 0.2 - 2 dB. In addition, it should be pointed out that the gain in the MS-5 system is achieved by an expensive modification to the apparatus.

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Evaluation of the Freedom from Interference of Communications Systems Which Use Opposite Phase-Manipulated Signals Under Conditions of the Action of Nonstationary Interferences

I.D. Zolotarev, B.N. Voronkov,
and A.P. Zhukov

Losses in signal/noise ratio at the output of a receiver for phase-manipulated signals which includes a unit to prevent powerful concentrations with respect to the interference spectrum were studied.

The use of a phase manipulated carrier to carry information means that the stability of the communications system can be greatly improved form the standpoint of the action of narrow band interferences. The unremovable part of the receiver in these systems is the shielding unit (BZ) for sinusoidal interferences. Since the frequencies of interferences are random, the shape of the frequency characteristic for the BZ cannot be predicted which, of course, lowers the freedom from static for the reception.

In this article an evaluation is made of the lowering of the freedom from interference for a receiver which consists of an optimum filter (OF) BZ, and a resolving device which operates on the principle of comparing polarities. The M-sequence is used as the rule for phase manipulation of the carrier. The information is transmitted by opposite signals. Because of this, in calculating the freedom from interference we must take into account not only the distortion of the envelope of the intercorrelation function (VKF) of the signal which passes through the BZ, but also the fine phase structure of the VKF. This complicates the calculation of the distortions for signals with large bases and makes it necessary to use a computer.

The BZ is built according to a "suboptimum" circuit [1]. It consists of M frequency channels which divide the signal's band into M segments. The transmission coefficient of each of the channels K_k is set in relation to the levels of the signal and interference in the corresponding frequency range. For a high enough interference level the corresponding frequency channel has a transmission coefficient equal to almost zero. In the following we will assume that the sinusoidal interference is completely suppressed (which is true when the selectivity of the filter in each of the channels is sufficiently good). In this case, a signal enters the input to the OF mixed with "white" noise which passes through the BZ, in which case the characteristics of the BZ are determined by the interference's frequency.

Now let us find the response of the OF to the signal which passes through the BZ. The complex transmission coefficient of the BZ can be written in the following form:

$$K_{\text{Б}3}(p) = \sum_{k=1}^{M} e^{\beta_k + j\varphi_k} \prod_{i=1}^{n} \frac{p + 2\alpha_{ik}}{(p + \alpha_{ik})^2 + \omega_{0ik}^2},$$

where M is the number of channels in the BZ.

$$e^{\beta_k + j\varphi_k} \prod_{i=1}^{n} \frac{p + 2\alpha_{ik}}{(p + \alpha_{ik})^2 + \omega_{0ik}^2}$$

is the transmission coefficient of the k-th channel, $e^{\beta_k + j\varphi_k}$ is a factor determined by the circuit which controls the transmission coefficients for the channels as a function of the interference, n is the number of resonance circuits in the channel, $\alpha_{ik}, \omega_{0ik}$ are the attenuation and frequency of the circuits in the k-th channel. The signal at the output of the OF is

$$U_{\text{вых}}(p) = K_{\text{Б}3}(p)\, K_{\text{ОФ}}(p)\, U_{\text{вх}}(p). \qquad (1)$$

where $U_{вх}(p)$ is the image of the phase-manipulated (FMn) signal $K_{БЗ}(p)$ is the BZ's transmission coefficient, $K_{ОФ}(p)$ is the transmission coefficient of the OF

As we know, $K_{ОФ}(p)\,U_{вх}(p)=R(p)$, $R(p)$ is the image of the autocorrelation function of the FMn signal:

$$R(p)=\sum_{m=0}^{N-1} b_m\left[\frac{p\sin\varphi_m+\omega_н\cos\varphi_m}{p^2+\omega_н^2}\,e^{-pm\tau_0}-\right.$$

$$\left.-\frac{p\sin\varphi_{m+1}+\omega_н\cos\varphi_{m+1}}{p^2+\omega_н^2}\,e^{-p(m+1)\tau_0}\right]e^{pt},$$

where N is the period for the M-sequence, b_m is the value of the grid autocorrelation function for the FMn signal, τ_0 is the duration of the elementary, discrete FMn signal, $\varphi_m=\varphi_н+m\,\omega_н\tau_0$, $\varphi_{m+1}=\varphi_н+(m+1)\,\omega_н\tau_0$, $\varphi_н$ is the initial phase and ω_H is the signal's carrier frequency.

By applying the method for the simplified inverse Laplace transformation [2] to (1) we get the VFK at the output to the OF:

$$U_{вых}(t)=\mathrm{Re}\left[\sum_{m=0}^{N-1} b_m\sum_{k=1}^{M}\left[\overline{A}_{mk}\,e^{j\omega_н t}\left[1(t-m\tau_0)-\right.\right.\right.$$

$$\left.-1\left[t-(m+1)\tau_0\right]\right]+$$

$$+\sum_{l=1}^{n}\overline{B}^l_{mk}\left\{\exp\left[(-\alpha_{lk}+j\omega_{0\,lk})(t-m\tau_0)\right]1(t-m\tau_0)-\right.$$

$$\left.-\exp(-\alpha_{lk}+j\omega_{0\,lk})\left[t-(m+1)\tau_0\right]\right]1\left[t-(m+1)\tau_0\right]\Big\}.$$

The coefficients $\bar{A}_{mk}$ and $\bar{B}^1_{mk}$ characterize the induced and free parts of the track's reaction. The noise's spectral density at the output of the OF is

$$N(\omega)=|K_{БЗ}(\omega)\,K_{ОФ}(\omega)|^2,$$

Thus, the noise at the OF's output is the sum of N realizations of the random process. In the first approximation we can assume that the shape of the correlation function for the noises in the region of the maximum changes only slightly. Therefore, the correlation function for the noises which enter into the output of the OF are determined basically by the frequency characteristic of the BZ

$$B_{\xi}(\tau) \approx \frac{1}{4\pi} \int_{-\infty}^{\infty} K_{\text{Б3}}^{2}(\omega)\, e^{j\omega\tau}\, d\omega = \sigma_{\xi}^{2} R_{\xi}(\tau),$$

where σ_{ξ}^{2} is the power of the noises at the output to the OF $R_{\xi}(\tau)$ is the correlation coefficient.

The treatment of the signals in the resolution circuit is carried out by the method of comparing the polarities in which it is assumed that the generator of the supporting frequency is not subjected to the effect of interference. The integration in the resolution circuit is done in terms of time T. The signal/noise ratio at the output of the integrator is determined by the formula [3]:

$$h^{2} = \frac{T\left[\frac{1}{T}\int_{0}^{T} U_{\max}(t) E_{\text{оп}}(t)\, dt\right]^{2}}{2\sigma_{\xi}^{2}\int_{0}^{T}\left(1-\frac{\tau}{T}\right) R_{\xi}(\tau)\, d\tau},$$

where $E_{\text{оп}}(t)$ is the supporting oscillation

Numerical calculations were made on a computer for an PM signal with N = 31. The number of periods in the sample was 4. The amplitude-frequency characteristics (AChKh) of the BZ for several types of interference interruption are shown in Figs. 1 and 2. The principal resultant lobes for the VFK of the signal which passes through the BZ are given, respectively, in Figs. 3 and 4. The signal/noise ratio at the output

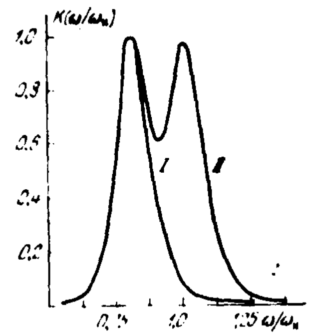

Fig. 1

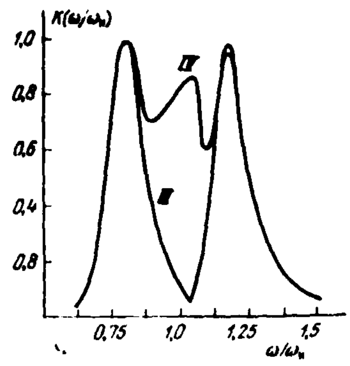

Fig. 2

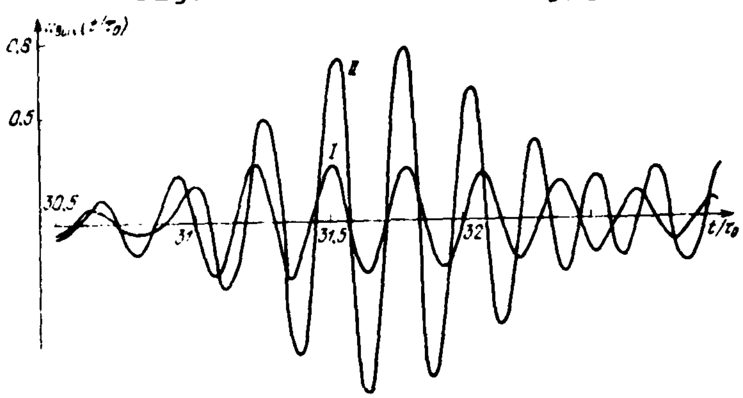

Fig. 3

to the resolution circuit was determined to be equal to the duration of the sample during the integration time. The coefficients which characterize the losses in the signal/noise ratio for the frequency characteristics of the BZ, which are shown in Figs. 1, and 2, are given below.

We can see from Figs. 3 and 4 that the integration time cannot be chosen arbitrarily since the strong correlation of the VKF of the signal, which passes through the BZ, with the various characteristics (which is due to the nonstationary nature of the interference) is only observed in a narrow time zone, close to the width of the VKF of the initial M sequence. However, the width of scattering for the VKF of the signal

Variations in the AChKh BZ	Loss coefficient
IV	1
II	0,9
I	0,12
III	0,064

which passes through the BZ, is determined by the value of the effective transmission band of the latter. If it is narrower than the signal's spectrum, then the difference between the permissible time of integration and the width of the VKF will be large. In this case the resolving circuit will only use a small part of the energy of the signal.

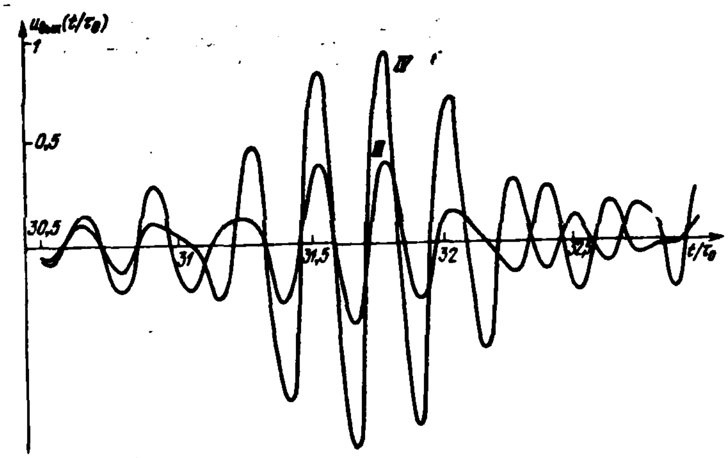

Fig.4

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Study of Systems with Phase Synchronization with a Finite Time for Data Collection

V.V. Shakhgil'dyan and V.A. Petrov

Linearized models are studied for systems with I and II order phase synchronization with a switch. A study of these systems can be used to determine the conditions necessary for the stability of their transitional and statistical characteristics.

Analysis of a linearized model for a system with phase synchronization with a switch. Taking into consideration the features of a radiopulse signal in a system with phase synchronization (FAPCh) additional elements are often introduced which improve the efficiency of its operation. Among these are a switch which is placed at the output of the phase detector to switch off the action of the noise during pauses and an extrapolator which, in the simplest case, fixes the value of the controlling voltage during the pause. The structural scheme for such a system when a signal with an initial frequency detuning and an additive noise acts on it is shown in Fig. 1 for the general case.

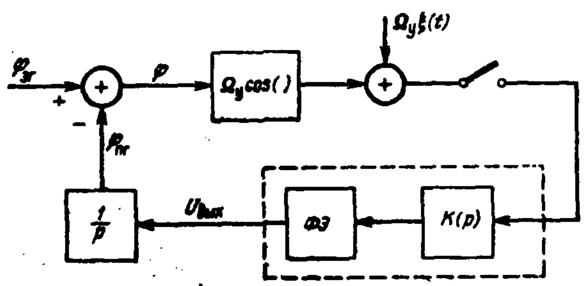

Fig. 1

The equations which describe the operation of the system during the action of the radio pulse and during the pause [1] can be given as

$$p\,\varphi + K(p)\,\Omega_y\,[\cos\varphi + \xi(t)] = \Omega_н; \quad t_k < t < t_k + h; \quad (1)$$
$$\varphi = \varphi_{эг} - U_{вых}(t)/p; \quad t_k + h \leqslant t \leqslant t_k + h + \theta = t_{k+1}. \quad (2)$$

here φ is the instantaneous phase difference between the incoming signal and that tuned by the generator, Ω_h is the holding band of the system, Ω_y is the initial frequency difference, K(p) is the transmission coefficient of the filter in the operative form, ξ (t) is the noise received at the output of the phase detector, h is the duration of the radio pulse, Θ is the duration of the pause, t_k is the moment for the beginning of the k-th radio pulse.

In the following we will discuss the operation of a FAPCh system with the simplest type of extrapolator

$$U_{вых\,фэ} = \begin{cases} U_{вх}(t); & t_k \leqslant t \leqslant t_k + h; \\ U_{вх}(t_k + h); & t_k + h \leqslant t \leqslant t_{k+1}. \end{cases} \quad (3)$$

For small deviations in the phase difference from the position of a stable equilibrium equation (1) can be given in the form

$$p\,\Delta\varphi + K(p)\,\Omega_y\,[(-\sin\varphi_{02})\,\Delta\varphi + \xi(t)] = 0, \quad (4)$$

where $\Delta\varphi$ is the deviation in the phase difference from the point of stable equilibrium as determined by the expression $\varphi_{02} = -\arccos\Omega_н/\Omega_y$.

If the error vector x is introduced, then Equation (1) and (2) can be written in the following matrix form [2] if we assume that the band for the noise at the input is much

greater than the system's band:

$$dx/dt = Q_1 x + r + f\xi^0(t), \quad t_k < t < t_k + h; \qquad (5)$$

$$dx/dt = Q_2 x + S x(t_k + h) + r, \quad t_k + h < t < t_{k+1}. \qquad (6)$$

where Q_1 and Q_2 are the quadratic matrixes of the coefficients, r is a vector associated with the external, determinant perturbations, $\xi^0(t)$ is white noise with a unit spectral density, f is a vector which characterizes the intensity of the noise, S is the matrix for the extrapolator (in the absence of a fixator S = 0).

On the basis of the solution for (5) and (6) an expression is found for the vector of the mean value m(t) and for the dispersion matrix D(t) in both intervals and, by substituting one into the other we get the recurrent equations [2].

$$m(t_{k+1} + h) = A\, m(t_k + h) + C; \qquad (7)$$

$$D(t_{k+1} + h) = AD(t_k + h) A^T + L, \qquad (8)$$

where

$$A = \Phi_1(h)\Phi_2(\theta); \qquad (9)$$

$$C = \Phi_1(h)\int_0^\theta e^{Q_1\tau} d\tau\, r + [I - \Phi_1(h)]\, m(\infty); \qquad (10)$$

$$L = D_1(\infty) - \Phi_1(h) D_1(\infty)\Phi_1^T(h); \qquad (11)$$

$$\Phi_1(t) = e^{Q_1 t} \qquad (12)$$

is a pulsed transition matrix for equation [5],

$$\Phi_2(t) = e^{Q_2 t} + \int_0^t e^{Q_2\tau} d\tau\, S \qquad (13)$$

is the pulsed transition matrix for equation (6), I is a unit matrix, $(\ldots)^T$ is a transposed matrix, $m(\infty)$ $D_1(\infty)$ are stationary values of the vector of mathematical expectation and the dispersion matrix determined from equation (5) [2]

$$m(\infty) = -Q_1^{-1} r; \qquad (14)$$

$$Q_1 D_1(\infty) + D_1(\infty) Q_1^T + B = 0. \qquad (15)$$

Here $B = ff^T$.

By solving (7) and (8) in terms of induction with $k \to \infty$ we can find stationary solutions which occur when the intrinsic numbers $\lambda_1, \ldots, \lambda_n$ of the matrix satisfy the condition [3, 4]

$$|\lambda_i| < 1. \tag{16}$$

In this case, the stationary values of the mean value and of the dispersion at the end of the pulse have the form [2]

$$\begin{aligned} m^{(k)} &= (I - A)^{-1} C; \\ D^{(k)} &= A D(\tau)^{(k)} A^{\tau} + L. \end{aligned} \tag{17}$$

For a II order system the characteristic equation for the matrix A has the form

$$\lambda^2 + B_1 \lambda + B_0 = 0,$$

where $B_1 = -\mathrm{Sp}\,A$; $B_0 = \det A$; SpA is the track of the matrix A.

By using the algebraic criteria [2], the conditions for the stability of a second order system can be written as

$$\begin{cases} \dfrac{1 - B_0}{1 + B_1 + B_0} > 0, \\ \dfrac{1 + B_0 - B_1}{1 + B_1 + B_0} > 0. \end{cases} \tag{18}$$

We will use this method in application to concrete systems with phase synchronization with a switch.

FAPCh system of the I order with a switch. The equations which describe the operation of the system during the action of a pulse and during a pause have the form

$$\Delta\dot{\varphi} = -\Omega_y\left[(-\sin\varphi_{02})\Delta\varphi + \xi(t)\right];\ t_k \leqslant t < t_k + h; \quad (19)$$
$$\dot{\varphi} = \Omega_n,\ t_k + h < t \leqslant t_{k+1}. \quad (20)$$

In the absence of noise their solutions at the end of the pulse and of the pause are the expressions:

$$\Delta\varphi(t_k + h) = \Phi_1(h)\ \Delta\varphi(t_k); \quad (21)$$
$$\Delta\varphi(t_{k+1}) \approx \Phi_2(\theta)\,\Delta\varphi(t_k + h) + \mathbf{r}\int_0^{\theta} e^{Q_2\tau}\,d\tau, \quad (22)$$

where
$$\Phi_1(h) = e^{Q_1 h} = e^{-\Omega_y(-\sin\varphi_{02})h}; \quad (23)$$
$$\Phi_2(\theta) = e^{Q_2\theta} = 1; \quad (24)$$
$$A = \Phi_1(h)\,\Phi_2(\theta) = \Phi_1(h); \quad (25)$$
$$Q_1 = -\Omega_y(-\sin\varphi_{02});$$
$$Q_2 = 0;$$
$$\mathbf{r} = \left\|\begin{matrix}0\\ \Omega_n\end{matrix}\right\|. \quad (26)$$

By finding the recurrent relationships from (21) and (22) and solving them in terms of the induction, we get expressions which can be used to determine the stationary values of the phase difference at the end $(\Delta\varphi_{CT}^{(h)})$ and at the beginning $(\Delta\varphi_{CT}^{(0)})$ of the pulse;

$$\Delta\varphi_{CT}^{(h)} = \frac{\Omega_n\theta e^{-\Omega_y(-\sin\varphi_{02})h}}{1 - e^{-\Omega_y(-\sin\varphi_{02})h}};$$
$$\Delta\varphi_{CT}^{(0)} = \Delta\varphi_{CT}^{(h)} + \Omega_n\theta. \quad (27)$$

The relationships (27) are valid for values of $\Omega_n\theta$, which satisfy the conditions of linearization.

Let us calculate the value of the stationary dispersion $D_{\varphi}^{(h)}$:

$$D_{\varphi}^{(h)} = A^2 D_{\varphi}^{(h)} + L\,, \tag{28}$$

where

$$L = D_{\varphi}(\infty)\,(1 - \Phi_1^2(h))\,. \tag{29}$$

The stationary value of the dispersion in the continuous system has the form

$$D_{\varphi}(\infty) = -\frac{B}{2\,Q_1} = \frac{G_0\,\Omega_y^2}{2\,U_{\text{эг}}^2\,\Omega_y(-\sin\varphi_{02})}, \tag{30}$$

where G_0 is the spectral density of the incoming noise at a zero frequency, $U_{\text{эг}}$ is the amplitude of the input signal.

By substituting (23), (25), and (30) in (29) and (28) we get

$$D_{\varphi}^{(h)} = D_{\varphi}(\infty)\,, \tag{31}$$

which coincides with the results found in [2].

<u>FAPCh of the I order with a switch and fixing element (FE)</u>. The equations which describe the work for such a system without noise have the following form:

$$\Delta\dot{\varphi} = -\Omega_y(-\sin\varphi_{02})\,\Delta\varphi; \quad t_k \leq t \leq t_k + h. \tag{32}$$

$$\Delta\dot{\varphi} = \Delta\dot{\varphi}(t_k + h); \quad t_k + h \leq t \leq t_{k+1}\,. \tag{33}$$

By carrying out the operations analogous to the case of a I order FAPCh with a key, we get the condition for the stability of the system in the following form:

$$e^{-\Omega_y(-\sin\varphi_{02})h} - |1 - h\,\Omega_y(-\sin\varphi_{02})| < 1. \tag{34}$$

For the presence of noise the dispersion in the phase error in this system will be equal to infinity (additive "white" noise). In such systems only the inertia of the fixator need be taken into account or the additive noise can be considered like filtered out "white" noise.

<u>FAPCh of the II order with a proportionally integrating filter (PIF) with a switch and fixing element at its output.</u> The equations which describe the opeation of a system in the absence of noise during the pulse and the pause can be written as

$$\begin{cases} \dot{x}_1 = x_2; \\ \dot{x}_2 = -\dfrac{\kappa}{T_1}x_1 - \dfrac{1+\kappa m T_1}{T_1}x_2; \ t_k \leqslant t \leqslant t_k + h: \end{cases} \tag{35}$$

$$\begin{cases} \dot{x}_1 = x_2(t_k + h); \\ \dot{x}_2 = 0; \quad t_k + h \leqslant t \leqslant t_{k+1}. \end{cases} \tag{36}$$

Here $\Delta\varphi = x_1$; $x_2 = \Delta\dot{\varphi}$; $\kappa = \Omega_y(-\sin\varphi_{02})$; $m_1 T_1 -$ is a parameter of the filter.

In this case, the matrix form for the recording (5) and (6) correspond to

$$Q_1 = \begin{Vmatrix} 0 & 1 \\ -\dfrac{\kappa}{T_1} & -\dfrac{1+\kappa m T_1}{T_1} \end{Vmatrix};$$

$$Q_2 = 0; \quad \mathbf{r} = 0; \quad S = \begin{Vmatrix} 0 & 1 \\ 0 & 0 \end{Vmatrix} \tag{37}$$

On the basis of (37) we can determine the pulsed, transition matrixes $\Phi_1(h)$ and $\Phi_2(\Theta)$ and also the matrix $A = \Phi_1(h)\Phi_2(\theta)$. Further, by finding the expressions for B_0 and B_1 we can calculate,

on the basis of (18) the region for the stability of the given system. As an example the results of calculating the stability of the system for different values of the duty factor C = T/h and for the parameter of the system m = 0.2 are given in Fig. 2.

As for the dispersion in the phase error, as in the preceding case, it will be equal to infinity.

FAPCh of the II order with a PIF and a switch for which capacitance of the PIF acts as a fixing element. The equations which describe the operation of the system during the pulse and the pause can be written as

$$\begin{cases} \dot{x}_1 = -m\,\Omega_y(-\sin\varphi_{02})\,x_1 + x_2 - m\sqrt{N_0}\,\xi^0(t); \\ t_k \leqslant t \leqslant t_k + h; \\ \dot{x}_2 = -\dfrac{\Omega_y}{T_1}(1-m)\,x_1 - \dfrac{1}{T_1}\,x_2 - \dfrac{(1-m)}{T_1}\sqrt{N_0}\,\xi^0(t); \end{cases} \tag{38}$$

$$\begin{cases} \dot{x}_1 = \dfrac{1}{1-m}\,x_2(t_k+h); \quad t_k + h \leqslant t \leqslant t_{k+1}; \\ \dot{x}_2 = 0, \end{cases} \tag{39}$$

where

$$x_1 = \Delta\varphi; \quad x_2 = -\frac{1-m}{T_1 p + 1} \times$$
$$\times\,\Omega_y(-\sin\varphi_{02})\,x_1 - \frac{1-m}{T_1}\sqrt{N_0}\,\xi^0(t);$$

N_0 is the coefficient which characterizes the intensity of the noise. By introducing $\tau = t/T_1$; $\dot{x}_2 = x_2 T_1$, equations (38) and (39) can be written in the matrix form (5) and (6)

$$Q_1 = \begin{Vmatrix} -mT_1\kappa & 1 \\ -(1-m) & -1 \end{Vmatrix}; \quad Q_2 = 0;$$
$$S = \begin{Vmatrix} 0 & \dfrac{1}{1-m} \\ 0 & 0 \end{Vmatrix}; \quad r = \begin{Vmatrix} 0 \\ 0 \end{Vmatrix}. \tag{40}$$

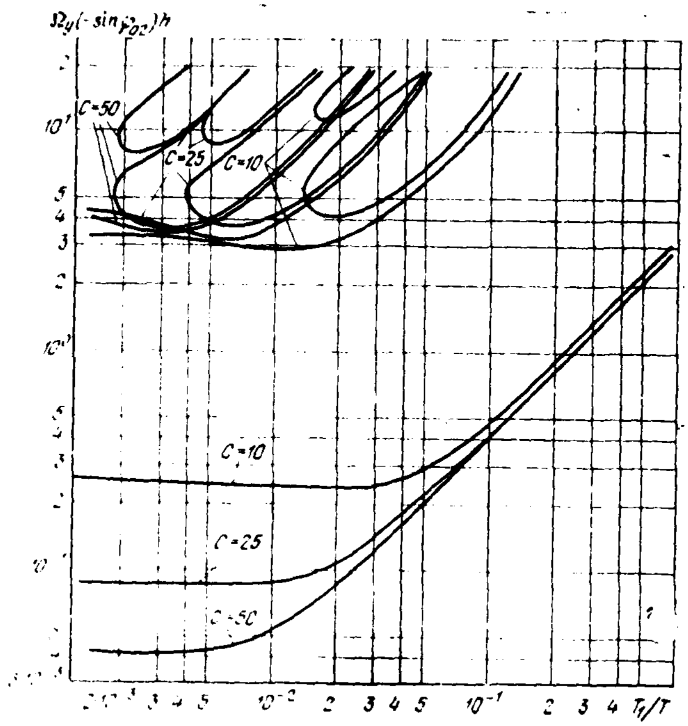

Fig. 2

On the basis of (40) we can determine the pulsed, transition matrices $\Phi_1(h/T_1)$, $\Phi_2(\theta/T_1)$ and the matrix A which allows us to calculate the range of stability for the system.

We can show that as a result of the calculation, the expressions being sought for B_0 and B_1, which determine the stability of the system, will coincide with analogous expressions for B_0 and B_1 in the preceding case.

Thus, the stability of the FAPCh system with a PIF and a switch does not depend on the fixed coordinate.

To calculate the dispersion in the phase error it is necessary to determine the correlation matrix B(15) and also the matrices $D_1(\infty)$ (15) and L (11). Then, by solving equation (17), we

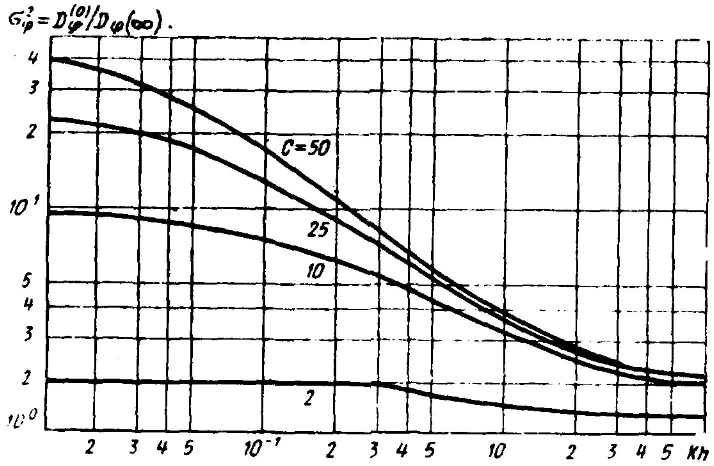

Fig. 3

can determine the matrix for the stationary dispersion at the end of the pulse $D^{(h)}$. The matrix for the stationary dispersion at the beginning of the pulse $D^{(0)}$ can be found from the expression

$$D^{(0)} = \Phi_2\left(\frac{\theta}{T_1}\right) D^{(h)} \Phi_2\left(\frac{\theta}{T_1}\right). \tag{41}$$

It should be pointed out that the calculation of the value for the stationary dispersion can be done more simply in numerical form by working with numerical matrices. The dependence of the relative stationary dispersion in the phase error at the end of the pause (value of the dispersion in a system with a switch, referred to the dispersion for a continuous system) on the duty factor (C) and for a system with the parameters $m = 0.2$; $T_1/T = 10$; $N_0 = G_0\,\Omega_y^2 / U_{\text{эт}}^2 = 0.1$. For m = 0 and C = 25 the results which were found coincide with the results in [2].

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Study of Digital Systems for the Automatic Phase Tuning of the Frequency

L.N. Belyustina, V.N.Belykh,
V.P. Maksakov, I.B. Petyashin,
and V.K. Perfilov

The derivation of the equations which describe the operation of a system is given. The stability and operating conditions are studied taking into account the presence of direct synchronization. The results are given of an experimental study of the given system.

In this work, a system with automatic phase tuning of the frequency (FAPCh) is studied with a discrete filter (DF) of the low frequencies taking into account the direct synchronization of the tuning generator (PG) with the standard generator (FG) (capture phenomenon). The structural scheme for such a system is shown in Fig. 1. The use of DF (Fig. 2) in the feedback circuit of the FAPCh was suggested in [1] and is due to the necessity of remembering the controlling voltage in the absence of a standard signal [2].

A mathematical model is set up by means of which the dependence of the capture band is determined on the parameters, the nature of the synchronous conditions, and the effect of direct synchronization with the standard generator on them. An experimental study was made of the synchronous conditions and of the system's capture bands.

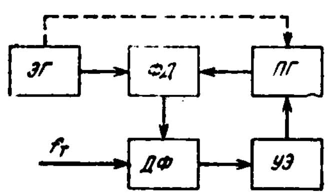

Fig. 1

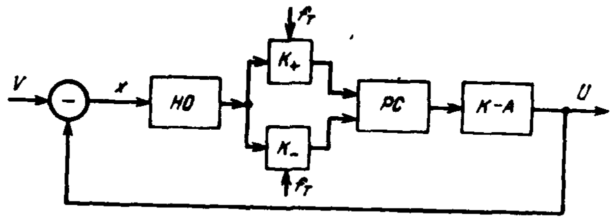

Fig. 2

Mathematical model. We will describe the given DF [3]. Let us assume that the zero element (NO) of the filter which controls the vents for the positive K_+ and negative K_- channels in the reverse counter (RS) has a part-constant characteristic of the type

$$R(x) = \begin{cases} 1 & \text{for } x > x_0, \\ -1 & \text{for } x < -x_0, \\ 0 & \text{for } |x| \leqslant x_0. \end{cases} \qquad (1)$$

Then, as the signal of uncoordination x = V-U passes through the boundary $|x| = x_0$ the time between two neighboring transitions t_i and t_{i+1} can be written in the form

$$n(t) = n(t_i) + R(x)\,\xi(t, t_i) \qquad (2)$$

where $\xi(t, t_i) = \left[\frac{t}{l}\right] - \left[\frac{t_i}{l}\right]$ (the brackets indicate the whole

or part of the number $l = \frac{1}{f_\tau}$, (3)

and $n(t_i)$ is the state of RS for $x(t_i) = |x_0|$. The value of $n(t_i)$ in (2) at the following moment of switching the NO ($t = t_{i\pm1}$) is determined by the recurrent relationship

$$n(t_{i+1}) = n(t_i) + R(x(t_i))\,\xi(t_{i+1}, t_i), \quad x(t_{i+1}) = |x_0| \qquad (4)$$

At the output the code-analog converter (K - A) forms a constant voltage U = n(t)δ , for which the voltage at the output for the DF has the form

$$U=[n(t_i)+R(x(t))\xi(t,t_i)]\delta. \qquad (5)$$

According to [4], if the direct action of the standard on it is taken into account, the equation for the tuning generator can be written in the form

$$d\varphi/dt+\Omega_0\sin\varphi+SU=\Omega_H, \qquad (6)$$

where Ω_0 is the range of capturing the PG by the standard, Ω_H is the initial detuning, and φ is the current phase difference of the standard and the PG.

Because of (5), upon introducing the new parameters $\Omega_y=SE$, $\gamma=\Omega_H/\Omega_y$, $m=\delta/E$, $\Omega_0/\Omega_y=\beta$, $\Omega_y l=h\,(E=\max u)$ and the dimensionless time $\tau=\Omega_y t$ equation (6) is converted to

$$\frac{d\varphi}{d\tau}=\gamma-\beta\sin\varphi-\left\{n(\tau_i)+R(x)\left(\left[\frac{\tau}{h}\right]-\left[\frac{\tau_i}{h}\right]\right)\right\}m. \qquad (7)$$

As a result we find that the given system is described by a sequence of equations (7) determined in the range $\tau_i-\tau_{i+1}$, where

$$R(x)=\begin{cases}1 \text{ for } \dot\varphi>\gamma+H-(1+\beta)\sin\varphi=f_+(\varphi),\\ -1 \text{ for } \dot\varphi<\gamma-H-(1+\beta)\sin\varphi=f_-(\varphi),\\ 0 \text{ for } f_-(\varphi)<\dot\varphi\leqslant f_+(\varphi),\ H=x_0/E. \end{cases} \qquad (8)$$

Because of the periodicity with respect to φ the boundaries f_+, f_- and the right hand side in (7) will be considered as a system of equations (7) (8) on the phase cylinder $(\varphi,\ \dot\varphi=y)$.

Let us designate a region on the cylinder (φ, y) in which R(x) is constant through $\Pi_+ : \dot{\varphi} > f_+(\varphi)$, $\Pi_- : \dot{\varphi} < f_-(\varphi)$ and $\Pi_0 : f_-(\varphi) < \dot{\varphi} \leq f_+(\varphi)$.

If the point which is formed is within the limits of one of the regions $\Pi_{+,0}$ in the period of time between two neighboring measured pulses h, equation (7) has the form

$$d\varphi/d\tau = r - \beta \sin \varphi, \quad r = \text{const.} \qquad (9)$$

This equation forms a group of integral curves on the cylinder (φ, y). For different values of $r \in (-\infty, \infty)$ along which the movement for the time t occurs. According to (7) at the moment the alternate pulse arrives the point which was expressed in the area $\Pi_+ (\Pi_-)$ completes its jump on the cylinder downwards (upwards) by a value m, i.e., it is converted into the integral curve (9) with $\bar{r}$ = r-m(r+m) and moves along this curve until the following pulse arrives. In the region of Π_0, the point which appears moves along one of the curves in the group (9) since at Π_0 R(x) = 0 the right hand side of (7) does not change either at the moment the pulses arrive.

This character for the movement on a cylinder can be used to reduce the problem to the study of a point representation of the cylinder on itself generated by the systems of equations (7) and (8). Let $\varphi = \Phi(t, r, \varphi_0)$ be the solution to equation (9) which satisfies the initial condition $\varphi_0 = \Phi(0, r, \varphi_0)$, when the point representation T, which relates any point M with the coordinates φ, y at the moment of time τ= ph with the following $\bar{M}(\bar{\varphi},\bar{y})$ at the moment of time τ = (p+1)h has the form

$$\left[\begin{array}{l} \bar{\varphi} = \Phi(h, r, \varphi), \\ \bar{y} = r - \beta \sin \Phi(h, r, \varphi), \end{array}\right. \quad r = \left\{\begin{array}{ll} y + \beta \sin \varphi, & M \in \Pi_0, \\ y - m + \beta \sin \varphi, & M \in \Pi_+, \\ y + m + \beta \sin \varphi, & M \in \Pi_-. \end{array}\right] \qquad (10)$$

In this way a mathematical model for the system is found in the form of a spliced point representation (10) of the cylinder

upon itself. When β = 0, i.e., when there is no direct synchronization, the form of the representation T is greatly simplified. In this case $\Phi(t, r, \varphi_0) = rt + \varphi_0$ and the respresentation (10) is converted into the form

$$
\begin{aligned}
T_+ &: \begin{cases} \bar{\varphi} = \varphi + (y - m)\,h, \\ \bar{y} = y - m \end{cases} && \text{for } M \in \Pi_+; \\
T_- &: \begin{cases} \bar{\varphi} = \varphi + (y + m)\,h, \\ \bar{y} = y + m \end{cases} && \text{for } M \in \Pi_-; \qquad (11) \\
T_0 &: \begin{cases} \bar{\varphi} = \varphi + y h, \\ \bar{y} = y \end{cases} && \text{for } M \in \Pi_0.
\end{aligned}
$$

Stationary conditions. According to the given model, the movement in a FAPCh system with DF is determined by the discrete trajectories in the representation (10), i.e., by the sequence of points $M^0, M^1 = TM^0, \ldots, M^k = T^k M^0$. When the integral is used

$$
t + C = \int \frac{d\varphi}{r - \beta \sin \varphi} =
$$

$$
= \begin{cases} \dfrac{8}{(r^2 - \beta^2)^{1/2}} \operatorname{arctg} \dfrac{r \operatorname{tg} 0{,}5\varphi - \beta}{(r^2 - \beta^2)^{1/2}} & \text{for } r^2 > \beta^2, \\[2ex] \dfrac{1}{(\beta^2 - r^2)^{1/2}} \ln \dfrac{r \operatorname{tg} 0{,}5\varphi - \beta - (\beta^2 - r^2)^{1/2}}{r \operatorname{tg} 0{,}5\varphi - \beta + (\beta^2 - r^2)^{1/2}} & \text{for } r^2 < \beta^2. \end{cases}
$$

Either the k=periodic, i.e., cycles which consist of such k point $M_1, M_2, \ldots, M_k$ from which the equality $T^k M_i = M_i$ holds for every M, or the quasiperiodic trajectories for each point of which M_j the inequality $|T^k M_j - M_j| < \varepsilon$. holds correspond to stationary conditions.

The condition which is determined by the periodic or quasi-periodic discrete trajectories $M^i(\varphi^i, y^i)$, for which the inequality

$$\max_{i,j} |\varphi^i - \varphi^j| < a < 2\pi; \quad \max_{i,j} |y^i - y^j| < b. \quad (12)$$

holds for their points is taken as a condition for synchronization of the FAPCh system with DF. All of the remaining stationary trajectories will be assumed to determine the asychronous conditions for the beat. It is apparent that among these are the trajectories which satisfy the inequality

$$a < \max_{i,j} |\varphi^i - \varphi^j| < 2\pi \qquad (13)$$

and which determine beats of the I type (oscillation type) and trajectories which do not satisfy (12) and (13) which determine beats of the II type (rotational type).

In this case the capture band is the range of changes in the initial detuning γ within the limits of which the synchronous conditions are established for almost any conditions. When the determination of the synchronized conditions (12) is introduced the capture band depends on the coefficients a and b. If the limitation (12) is not needed the "expanded" capture band for $a = 2\pi$ must be kept in mind.

Let us consider what types of concrete stationary conditions which are realized in a FAPCh system with a DF.

1. When there is no direct synchronization ($\beta = 0$), each of the representations which enter into (11), T_+, T_-, and T_0 has a group of integral curves L_+, L_-, and L_0 of the type

$$L_+:\ \varphi = C_+ - \frac{h}{2m}\left(y - \frac{m}{2}\right)^2,$$
$$L_-:\ \varphi = C_- + \frac{h}{2m}\left(y + \frac{m}{2}\right)^2,$$
$$L_0:\ y = C_0,$$

in which C_+, C_-, and C_0 are constants. It follows from this that the representations T_+ and T_- do not have stationary points and periodic trajectories and the representation T_0 has two segments $A(y=0,\ \varphi<\pi/2,\ \varphi\in\Pi_0)$ and $B(y=0,\ \varphi>\pi/2,\ \varphi\in\Pi_0)$ for the stationary points on which the point of the spliced representation which is expressed (11) can only fall within the exclusively initial conditions, viz., $y_{нач} = km$ (k is an integer). Thus, the basic stationary conditions in the system are determined by periodic (or quasiperiodic) trajectories whose every point is stationary with respect to the representations of the type $\tilde{T} = \prod_{i=1}^{N} T_+^{p_i} T_0^{n_i} T_-^{q_i}$. The trajectories which determine the stationary conditions on the part of the cylinder φ, y for different values of the parameters, which were found on a computer are shown in Fig. 3. Fig. 3, a and b, illustrate the periodic trajectories and Fig. 3c, illustrates the quasi-periodic trajectory (with an accuracy of $\varepsilon = 10^{-7}$). The sequency of the trajectory points is numbered. The action of the representation T_0 on the straight lines y = const in the areas between the wavy lines is not shown in the figures. The oscillograms of the same stationary process (dependence of the DF's output on U(t) which corresponds to synchronized conditions and which were found experimentally for three different intervals of time are shown in Fig. 4 a, b, c. We can see from these figures that the sequency of representations $T_+T_0T_-$ for either a periodic trajectory with a large period or a quasiperiodic trajectory corresponds to such a complicated condition.

2. When direct synchronization is taken into account, i.e., for $\beta > 0$, the mathematical model for the system is the representation (10). The coefficient β in (10), which determines

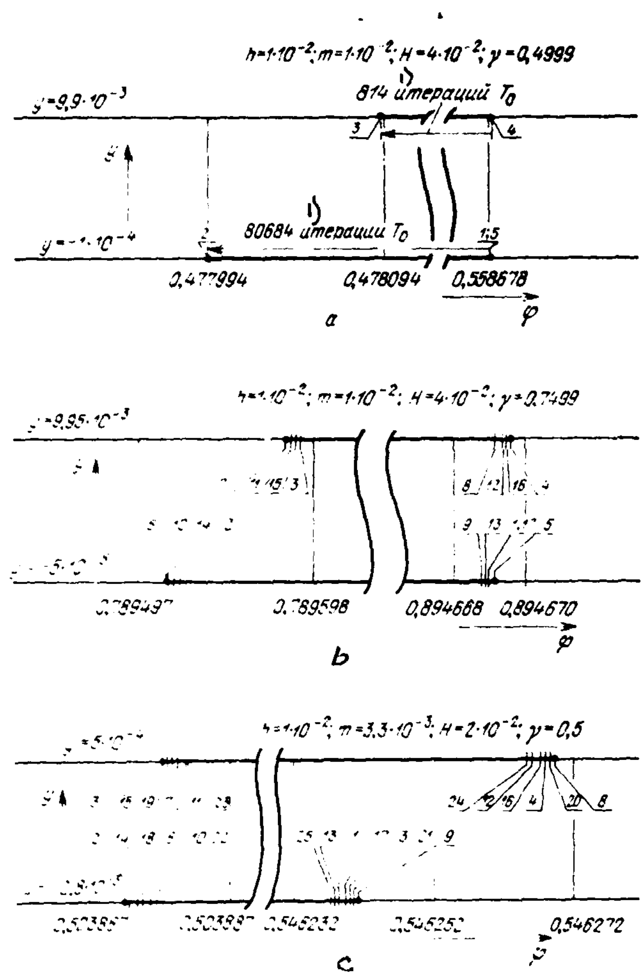

Fig. 3

1) iteration

the capture of the PG by the standard generator, is assumed to be small since usually $\Omega_0 \ll \Omega_y$, For $\beta \ll 1$ the character of the movement for the points in the representation (10) remains the same as for representation (11) except for the one singularity which appears in this case. It consists of the fact that the segment A, for representation (10) becomes the asymptotically stable segment for the stationary points and the segment B becomes the unstable segment for the stationary

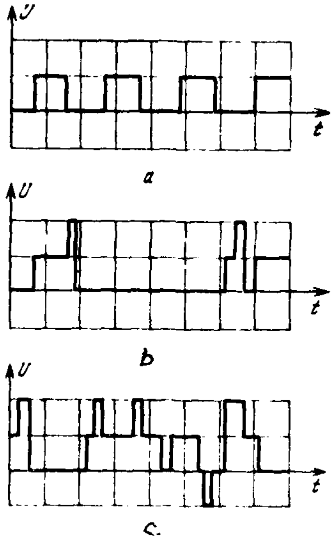

Fig. 4

points. Thus, for $\beta \neq 0$, in contrast to (11) precise synchronization, which is determined by the stable segment A, is not exclusive and is realized for the initial conditions for their region of attraction of the segment A, regardless of their multiple of the number m. Because of the appearance of a region of attraction for segment A as β is increased from zero at first part of the periodic trajectories and then, possibly, all of the periodic trajectories (for example of $\beta = 1$) disappear. Thus, for small values of β the stationary conditions of synchronism are determined by the stable segment for the stationary points A and by the periodic movements which "capture" it and for large values of β they are determined possibly, by a single segment A.

The complex form of the representation (10) makes its qualitative study difficult but it does not prevent its being studied on a computer.

In conclusion, let us consider the operation of the system under conditions of memory [2], i.e., when the cyclic pulses are switched off when the incoming signal arrives. Let the system be under synchronized conditions when the cyclic pulses are switched off. This means that the point which is represented in this case is either on the segment A or it is moving rotationally in the area $\Delta\varphi<a$, $\Delta y<\dot{b}$, determined by the inequalities (12). When the cyclic pulses are switched off for a time $\Delta\tau$ the function $\xi(t, t_1)$ in (12) does not change and, consequently,

the system during the period of time $\Delta\Upsilon$ is described in the region Π_+ and Π_- by the same equations as in the ration Π_0.

Thus, if the point which is represented is on the segment A then the system during the time of disconnection $\Delta\Upsilon$ remains in the same position and if it is rotating then in the period of time $\Delta\Upsilon$ the point which is represented can move along φ by a maximum value of $b\Delta\Upsilon$. When the cyclic pulses are switched on the movement in the system is described by the representation (10) and their disconnection is the same as changing the initial conditions with respect to φ by a value which does not exceed $b\Delta\varphi$.

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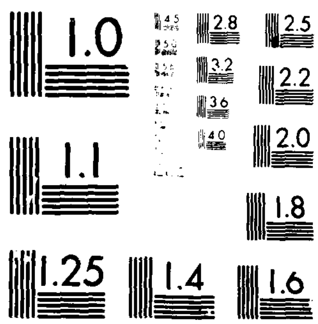

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Efficiency of Systems for Transmitting Discrete Signals

V.L. Banket

The efficiency of using the frequency bands and signal's power for systems for the transmission of discrete signals is studied. Maximum values are given for the efficiency index for systems with binary and multilevel signals. The results of the calculations of the efficiency of systems which are used in practice are compared with the maximum values calculated for the ideal system.

As systems for the transmission of signals through satellite relays were developed, the increase in the efficiency for the transmission of discrete communications along channels with constant parameters and fluctuation interference has again acquired added significance. In the first stage for the development of such systems particular attention was given to the energy indices. However, even now, the economical use of the frequency band of the satellite trunk becomes more and more important. The systems which are in use today are built, mainly, on a basis of signals which are simple in shape and which have rather low efficiency indices. However, with the development of circuitry and micro-electronics it is possible to convert to systems with more complex signals which give higher efficiency indices.

The efficiency of a system for the transmission of discrete communications was studied in [1-4]. The following indices have been used most widely:

$$\eta = R/C \qquad (1)$$

is the efficiency of using the transmitting capacity of the channel C, R is the rate of transmission of the communications up the channel,

$$\beta = \frac{R}{P_c / N_0} \qquad (2)$$

is the efficiency of using the power of the signal P_c, transmitted on a background of fluctuation noise with an even spectral desnity N_0,

$$\gamma = \frac{R}{F} \qquad (3)$$

is the efficiency of using the frequency band of the channel F.

The index

$$\beta = R N_0 F / P_c F = \gamma / q. \qquad (4)$$

where $q = P/N_0F$ is the signal/noise ratio measured in the channel's band F.

Systems can be compared in their efficiency by analyzing the individual indices, however, the most complete comparison is that made by using the $\beta\gamma$ diagram in which the individual systems are represented in the form of points (Fig. 1). The representation of the system in the coordinates β and γ is the more convenient because the dependence $\beta = f(\gamma)$ can be found for an ideal system on the basis of the most general relationships of information theory.

The transmission capacity of a channel for the action of "white" noise is equal [5] to

$$C = F \log_2(1 + q). \qquad (5)$$

By assuming, for an ideal system that $C = R$ and by using equations (2) and (3), it is not difficult to find the dependence (Fig. 1)

$$\beta = \gamma / (2^{\gamma} - 1). \tag{6}$$

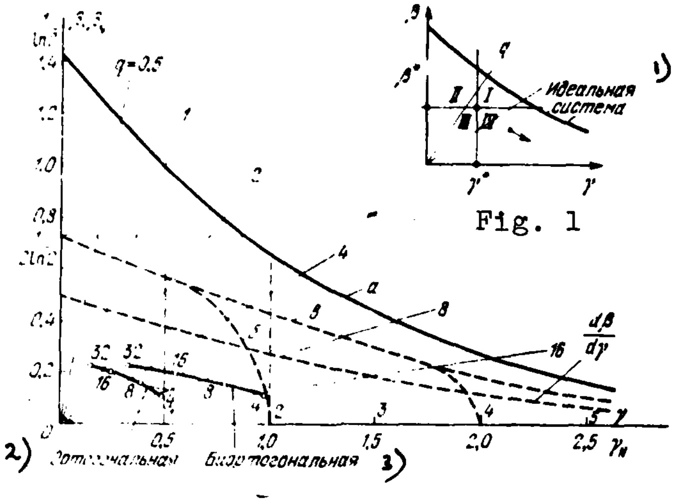

Fig. 2
1) Ideal system, 2) orthogonal, 3) biorthogonal

Real systems with the indexes β and γ less than the maximum values are in the region below this curve. The lines with the same signal/noise ratio, q, are shown by the sloped straight lines. In the usual case the diagram can be used for a quick evaluation of the possibilities of various systems. For example, for given transmission rates R*, the frequency band for the channel F* and the signal/noise ratio q* for the entire range of values of $\beta\gamma$ can be divided into four quadrants. Systems which satisfy the requirements $\beta>\beta^*$ and $\gamma>\gamma^*$, are in the first quadrant. Systems which are in the II and IV quadrants do not satisfy the requirements for β and γ, respectively, and systems in the III quadrant do not satisfy either of these indices.

The curve in Fig. 1 shows that it is possible to replace the β efficiency by the γ efficiency. This exchange appears to be nonequivalent. In Fig. 2 the curve for the derivative function (6) is

$$\frac{d\beta}{d\gamma} = \frac{(2^{\gamma} - 1) - 2^{\gamma}\gamma \ln 2}{(2^{\gamma} - 1)^2}, \qquad (7)$$

is shown by the dotted line. It follows from this curve that for small values of γ the exchange conditions are the best. A reduction in the β efficiency can give as high a γ efficiency as desired. However, reducing γ does not give as great an increase in β as desired since, from equation (6) it follows that $\lim_{\gamma \to 0} \beta = 1/\ln 2$.

The range for the existence of possible systems for the transmission of communications can conditionally be divided into two parts: systems with a high β (but small γ) and, vice versa, systems with a high γ (and a correspondingly low β). Systems in which the energy indices are most important and in the second - those for which the frequency band is most important.

It should be pointed out that representing real communication transmission systems by means of the $\beta\gamma$ diagram is approximate. The freedom of the system from static, and consequently, the β efficiency, in the majority of cases can be determined accurately only if the linear distortion of the signal, introduced by the limitation of the communications channel's band, are not taken into account. Thus, the position of the point being represented for a given system along the vertical (β-axis) can be placed accurately but without taking into account the values of γ. A limitation on the band results in distortions of the signals and it decreases β. This reduction is the greater the lower the band frequency which is used for a given rate R, i.e., the larger the value of γ. In this case a typical movement of the point being represented for a real system will be like that shown in Fig. 1 by the arrow.

It is usually assumed that the transmitted communication occupies a frequency band F and is transmitted in the time T. However, the concept of a frequency band, in contrast to other indexes, cannot always be found accurately. The distortion of the signal is determined to a large extent by the type of transmission coefficient for the track also. For precise calculations it is better to assume that a certain number of independent coordinates (readings) N are removed for transmission [7] which, in turn, can be determined from the track's transmission index

$$N = kFT. \qquad (8)$$

The coefficient k depends on the characteristic of the transmission track and its shows what number of readings can be had in a unit of the band in a unit of time. Then the number of readings, used to transmit for a unit of time is

$$D = kF, \qquad (9)$$

and the total number of readings is N = DT.

Let us designate the specific rate of transmission as R_N = R/D, i.e., the amount of information in one reading in a unit of time. In this case the efficiency index will have the form

$$\eta = RD/DC = R_N/C_N = \eta_N, \qquad (10)$$

where C_N = C/D is the specific transmitting capacity of the channel,

$$\beta = \frac{R}{P_c/N_0} = \frac{R_N D}{P_c} N_0 = \frac{R_N}{E_N/N_0} = \beta_N, \qquad (11)$$

where $E_N = P_c/D$ is the energy of the signal which is received in one reading,

$$\gamma=\frac{R}{F}=\frac{RD}{DF}=R_N k=\gamma_N k. \qquad (12)$$

In this case the formula for the transmitting capacity of the ideal communications system for N = 2FT has the form [6]:

$$C_N=\frac{1}{2}\log_2\left(1+\frac{2E_N}{N_0}\right), \qquad (13)$$

from which we can readily find the expression for the efficiency $\beta_N = 2\gamma_N/2^{2\gamma_N}-1$. The dependence $\beta_N = f(\gamma_N)$ is shown in Fig. 2 (curve "a").

In terms of the geometric theory of signals, a system which has the transmission capacity (13) is characterized by the optimum distribution of the signal points within a certain spherical volume (optimum packing of the signal range). Therefore, any real methods for packing the signal range have a lower efficiency and are represented by points which lie beneath the curve $\beta_N = f(\gamma_N)$.

The maximum efficiency of systems in which certain limitations are placed on the method of building the signals is of interest. For example, in practice, systems are widely used with binary signals (binary sequences). In this case, the evaluation of the average probability of error has the form [6]

$$\bar{P}_{er}<2^{-N(R_0-R_N)}, \qquad (14)$$

where $R_0 = 1-\log_2(1+e^{-E_N/N_0})$.

From (14) we get the expression for the efficiency

$$\beta_N = -\gamma_N/\ln(2^{1-\gamma_N}-1). \qquad (15)$$

The curve $\beta_N = f(\gamma_N)$ is shown in Fig. 2 (curve "b"). We can see that the limitation of the transmitted signals to only binary sequences greatly contracts the possibilities of the system. In this case, the maximum value of γ_N is 1.

The use of binary sequences corresponds to a surface packing of the signal range in a sphere of fixed radius. The band readings can only be used better for transmission methods which allow the information load on each reading of the signal to be increased without increasing the number of readings in the initial band. The method for the amplitude modulation of the readings, which gives the optimum placement of the signal range on spheres of ever increasing diameter with a common center, is such a method. In this case the transmission is carried out by multilevel sequences and the signals have different energies. An expression was given in [6] for the coefficient for such signals

$$R_0 = \frac{\log_2 e}{2}\left[1 + \frac{E_N}{N_0}\sqrt{1 + \left(\frac{E_N}{N_0}\right)^2}\right] + \\ + \frac{1}{2}\log_2\left[\frac{1}{2}\left(1 + \sqrt{1 + \left(\frac{E_N}{N_0}\right)^2}\right)\right]. \qquad (16)$$

The values of the points on the curve $\beta_N = f(\gamma_N)$, which are found from equation (16) are shown in Fig. 2 (curve "c').

It follows from this diagram that the number of transmission levels which it is expedient to use is a function of q. Specifically, for q $\leq$ 2 the conversion from binary to multilevel transmission does not increase the efficiency of the system.

Often orthogonal, biorthogonal, simplex, disimplex, and other signals are used for the transmission of discrete communications.

The probability of error for orthogonal signals [6] is

$$P_{ош} \leqslant \exp\left[-N\ln 2\left(\frac{E_N}{2\ln 2 N_0}-R_N\right)\right], \qquad (17)$$

from which it is easy to get the following condition for the efficiency

$$\beta_N \leqslant 1/2 \ln 2. \qquad (18)$$

On the other hand, the number of orthogonal signals M cannot exceed the number of readings N($M \leqslant N$), i.e., the maximum value of the efficiency of using the readings will be

$$\gamma_N \leqslant \log_2 N/N, \qquad (19)$$

where $\gamma_N \leqslant 0.5$ for N = 2, $\gamma_N \to 0$ for $N \to \infty$. For biorthogonal signals $\gamma_N \leqslant 1$. These two conditions determine the range of positions for systems with orthogonal (or biorthogonal) signals.

The limiting curves are shown in Fig. 2 which determine the range of the positions for signals with different properties. The singularity of these curves is that they are set up for systems in which the probability of error can be reduced to as low a value as desired by increasing N. It is characteristic that for $N \to \infty$ the packing of the signal range approaches the ideal value.

Real systems are characterized by a finite value of N and a finite value of $P_{ош}$. This results in a nonoptimum placement of the signal range (nonoptimum packing) and to a reduction of the efficiency index as compared with the ideal value.

The results of calculations of the efficiency for different systems with a different number of positions for the signal

are shown in Fig. 2. The calculations were made of $P_{ош} = 10^{-5}$. We can see that the signal systems which are used in practice have a relatively low efficiency as compared with the ideal efficiency.

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Selection of Filters for Systems with Phase Modulation

P.V. Ivashckenko

The parameters were found for the optimum filters for systems with phase manipulation and a band-limited channel. A method is described for calculating the energy losses for using other than optimum filters.

We know that in the case of a double channel the greatest freedom from interference is provided by opposing signals: $s_2(t) = -s_1(t)$.

In view of the complexity of controlling a multichannel system with synchronized channels, systems with independent channels are often used in practice. In this case, the channels can be separated by using filters which limit the signals' spectra. The communications channel, in this case has the form shown in Fig. 1.

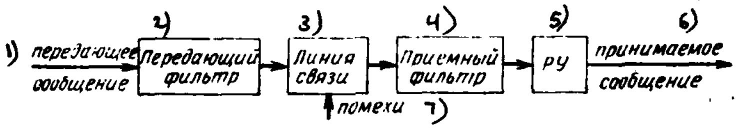

Fig. 1

1) transmitted communication, 2) transmitting filter, 3) line of communication, 4) receiving filter, 5) RU, 6) received communication, 7) interference

The transmitting filter is stimulated by a sequence of δ-pulses of different polarity which correspond to the symbols of the transmitted communication and which follow one another at intervals of time Δt. The signal is a pulsed response of the filter and the signal's spectrum is completely determined by the filter's parameters.

$$S(j\omega) = K_{\text{пер}}(j\omega). \quad (1)$$

The response for a decrease in the interference-free nature of the reception are:

1) Lack of coordination between the receiving filter and the signal which means that the maximum signal/noise ratio cannot be attained at the moment of reading at the input to the resolving device (RU).

2) The presence of intersymbol interference at the moment of time for reading at the input to the RU.

3) The presence of intermediate interferences from neighboring channels.

The effect of these reasons is determined by the characteristics of the transmitting and receiving filters. Moreover, there are a number of factors which lower the freedom from interference which either depend only slightly or not at all on the filters: fluctuations in the moments of sustaining the resolution, lack of precision for the RU because of the finite resolving capacity and the dynamic range, inaccuracy of the phase for the supporting oscillation in the demodulator in transmitting the signals along the radio channel.

In this work the effect of the first group of reasons was studied on the value of the energy losses in a communications system.

Further, by assuming that the radio signals are narrow band, we will use the signal's envelope, and the spectra of the envelopes and the low frequency equivalent circuits.

Let $K_{пр}(j\omega)$ be the transmission coefficient of the receiving filter, $K_{л}(j\omega)$ be the transmission coefficient of the line of communications, $N(\omega)$ be the energy spectrum of the interference at the input of the receiving filter.

The signal's spectrum at the input to the RU is

$$S_{\text{Py}}(j\omega) = K_{\text{пер}}(j\omega)\, K_{\text{л}}(j\omega)\, K_{\text{пр}}(j\omega). \qquad (2)$$

The condition for maximizing the signal/interference ratio at the input to the RU is [1]

$$K_{\text{пр}}(j\omega) = \frac{K^{*}_{\text{пер}}(j\omega)\, K^{*}_{\text{л}}(j\omega)}{N(\omega)}\, e^{-j\omega t_0}, \qquad (3)$$

where t_0 is the delay selected from the conditions for realizing $K_{\text{пр}}(j\omega)$.

In the absence of intersymbol interference the voltage at the input to the RU must have the form

$$b(k\,\Delta t) = \begin{cases} b_0, & k = 0, \\ 0, & k \neq 0,\ k = \pm 1, \pm 2, \ldots \end{cases} \qquad (4)$$

Here the moment which corresponds to the maximum value for b(t) is taken at the beginning of the reading which does not lower the general nature of the discussion.

It was shown in [2, 3] that the function of the type (4) corresponds to the Nyquist spectrum. Consequently, the Nyquist spectrum must also hold at the input to the RU $S_{\text{Py}}(j\omega)$. A comparison of equations (2) and (3) shows that the spectrum $S_{\text{Py}}(j\omega)$ is additive and according to [2]

$$S_{\text{Py}}(\omega_c + \omega) + S_{\text{Py}}(\omega_c - \omega) = \text{const},$$

where $\omega_c = \pi/\Delta t$; ω is the current frequency.

The phase characteristics of the filters can be arbitrary to a certain extent - it is sufficient that the following equality be fulfilled

$$\varphi_{\text{пр}}(\omega) = -[\varphi_{\text{пер}}(\omega) + \varphi_{\text{л}}(\omega)] + \omega t_0.$$

We can write the following for the moduli of the filter's transmission coefficients, on the basis of equations (2) and (3).

$$|K_{пр}(j\omega)| = \sqrt{S_{py}(\omega)/N(\omega)},$$

$$|K_{пер}(j\omega)| = \sqrt{S_{py}(\omega)\,N(\omega)}/|K_{л}(j\omega)|.$$

In the specific case of an even spectrum the interferences $N(\omega) = N_0$ and $K_{л}(j\omega) = K_{л0}$, are determined with an accuracy up to the constant multiple of the AChKh filters

$$|K_{пер}(j\omega)| = |K_{пр}(j\omega)| = \sqrt{S_{py}(\omega)}. \qquad (5)$$

Three specific cases for the case of a Nyquist spectrum are given in Fig. 2. $S_{py}(\omega)$ should be selected in the form of AChKh of the ideal FNCh (Fig. 2a) to get a high efficiency when using the frequency band.

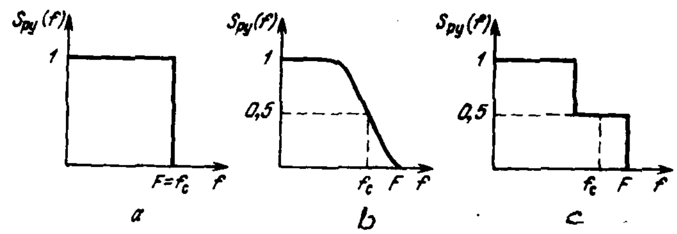

Fig. 2

In this case, the frequency f_c is equal to the channel band F and the coefficient of using the frequency band γ = 2 dB units/Hz. For an obliquely symmetrical slope for the Nyquist amplitude spectrum $\gamma < 2$ dB units/Hz (Fig. 2b). If the condition $f_c < F$ is fulfilled in the system then, as was shown in [3], the Nyquist amplitude spectrum should have the shape given in Fig. 2c. This type of spectrum allows us to maximize b_0 at the moment of reading for a fixed signal energy.

Circuits which give the spectrum shown in Fig. 2 can only be built with a finite degree of accuracy. In this case condition (4) is not obeyed and at the moment of time of the reading the voltage for the intersymbol interference from the preceding and following symbols exists at the input to the RU and the wanted voltage of the signal does not reach its maximum value because the receiving filter is not coordinated with the signal.

If the receiving filter is coordinated with the signal then the ratio for the maximum voltage at the output to the filter U_M to the effective voltage for the noise $\sigma_{\text{ш}}$ is determined by the known equation [1]

$$(U_{\text{м}}/\sigma_{\text{ш}})^2_{\text{макс}} = 2\,E/N_0,$$

where E is th signal's energy at the input to the filter. According to Parseval's theorem

$$E = \frac{1}{\pi}\int_0^\infty |S(j\omega)|^2\,d\omega.$$

Taking equation (1) into account we find the signal's energy

$$E = \omega_{\text{эф1}}/\pi,$$

where $\omega_{\text{эф1}}$ is the effective band for the receiving filter.

In the usual case

$$\left(\frac{U_{\text{м}}}{\sigma_{\text{ш}}}\right)^2 = \frac{U_{\text{м}}^2}{N_0\,\omega_{\text{эф2}}/2\pi},$$

where $\omega_{\text{эф2}}$ is the effective band of the receiving filter and the energy losses

$$\theta = \frac{(U_{\text{м}}/\sigma_{\text{ш}})^2_{\text{макс}}}{(U_{\text{м}}/\sigma_{\text{ш}})^2} = \frac{\omega_{\text{эф1}}\,\omega_{\text{эф2}}}{(\pi\,U_{\text{м}})^2}. \qquad (6)$$

The value of U_M can be found from the response of the receiving filter.

Interference-free coherent reception under condition with intersymbol interference in receiving the i-th combination is determined by the probability of error

$$P_{oi} = V\left(\frac{U_м + u_{мci}}{\sigma_{ш}}\right), \tag{7}$$

where

$$V(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt$$

is the integral for the probability, $u_{мci} = \sum_{l=1}^{n} \xi_l\, b(l\,\Delta t)$— is the total voltage of the interfering responses at the input to the RU at the moment of reading, ξ = ±1 are the interfering responses which are taken into account (preceding and following) for the reception of the i-th combination, and n is the number of responses which are taken into account.

The total probability of error in receiving the signal is

$$P_o = \frac{1}{2^n} \sum_{i=1}^{2^n} P_{oi}, \tag{8}$$

where P_{oi} is determined from (7) for all of the possible combinations of values of n.

We will use the method given in [4] to calculate the energy losses taking into account the intersymbol and interchannel interferences. The mean probability of error in receiving a signal in the i-th combination for the action of interchannel interference is

$$\overline{P}_{oi} = P_{oi} + \sigma^2_{мк} P''_{oi}/2,$$

where $\sigma^2_{\text{мк}}$ is the dispersion for the voltage of the interchannel interference and P''_{ot} is the second derivative of the probability of error

$$\sigma^2_{\text{мк}} = \frac{2}{\pi} \sum_{m=1}^{M} K^2_{\text{пр}}(\omega) \left[K^2_{\text{пер}}(\omega + m\,\Delta\,\omega) + \right.$$
$$\left. + K^2_{\text{пер}}(\omega - m\,\Delta\,\omega) \right] d\,\omega, \qquad (9)$$

where $\Delta\omega$ is the detuning between the channels, M is the number of interfering signals which are taken into account (from one side of the main channel) $K^2(\omega)$ is the square of the modulus of the transmission coefficient for the filters being used

$$P''_{ot}(v) = \frac{1}{\sqrt{2\pi}} \frac{v}{\sigma^2_{\text{ш}}} e^{-v^2/2\sigma^2_{\text{ш}}}.$$

The final expression for the total probability of error, taking into account the intersymbol and interchannel interference assumes the form

$$P_o = \frac{1}{2^n} \sum_{i=1}^{2^n} \left[V\left(\frac{U_{\text{м}} + u_{\text{мс}i}}{\sigma_{\text{ш}}} \right) + \right.$$
$$\left. + \frac{1}{2\sqrt{2\pi}} \left(\frac{\sigma_{\text{мк}}}{\sigma_{\text{ш}}} \right)^2 \frac{U_{\text{м}} + u_{\text{мс}i}}{\sigma_{\text{ш}}} e^{-\left(\frac{U_{\text{м}} + u_{\text{мс}i}}{\sigma_{\text{ш}}} \right)^2} \right]. \qquad (10)$$

As an example, let us consider the use of filters in the modulator and demodulator of the third order polynomila type whose transmission function is

$$K(p) = \frac{p_1\, p_2\, p_3}{(p - p_1)(p - p_2)(p - p_3)},$$

where p_1, p_2, and p_3 are the bands of the transmission function.

We will assume that the transmitting and receiving filters are the same in this case, because (5) is fulfilled. The receiving

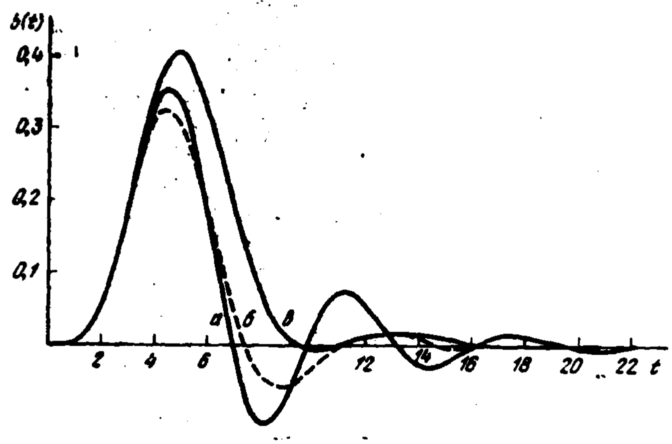

Fig. 3

filter is closest to a coordinated filter. The response at the input to RU at b(t) can be found as the inverse Laplace transformation of the product of the transmission functions for the transmitting and receiving filters. As an illustration the responses b(t), calculated for Chebyshev filters (a), Batterworth filters [5] (b) and with a leveled off group time [6] (c) are shown in Fig. 3. The function b(t) is given in the scale of normalized time

$$t = t_{сек}\,\omega_0,$$

where ω_0 is the frequency with reference to which the normalization was done.

The results of the calcualtion of the energy losses by means of equation (6) are given in the table for $\omega_{эф1} = \omega_{эф2}$. Here ΔT is the normalized maximum deviation of the group time for the filter in the band, $|K(j\omega)|_{мин}$ — is the minimum transmission coefficient in the band which characterizes the wave properties of the AChKh.

The energy losses due to the lack of coordination between the receiving filter and the signal are found as a direct function

of the nonlinearity of the FChKh of the filters that are used. The losses increase greatly in converting to filters with a higher selectivity.

Using equations (7), (8) and the graph for b(t) (Fig. 3) the dependence $P_0(b_0/\sigma_n)$ are calculated for different values of Δt, which can be used to find the dependence of the energy losses which occur because of the intersymbol interference on the interval Δt (Fig. 4). We can see that the energy losses in using Chebyshev filters is much greater than that for using Batterworth filters.

Type of filter	filter parameters	Θ, dB
With leveled group time	$\Delta T = 0{,}2$	0,06
	$\Delta T = 0{,}4$	0,14
Batterworth		0,19
Chebyshev	$\|K(j\omega)\|_{\text{мин}} = -0{,}5$ дБ	0,35
	$\|K(j\omega)\|_{\text{мин}} = -1$ дБ	0,44
	$\|K(j\omega)\|_{\text{мин}} = -2$ дБ	0,64

Usually the rate of telegraphing V and the dispersion between the channels Δf are usually given. Assuming that the normalized time $\Delta t = \omega_0/V$ is the normalized dispersion

$$\Delta\omega = \frac{2\pi\Delta f}{\omega_0} = 2\pi\frac{\Delta f}{V}\frac{1}{\Delta t}. \qquad (11)$$

By setting $\Delta t = \Delta t_{\text{опт}}$ in (11) for which the minimum loss is attained because of the intersymbol interference, we can find $\Delta\omega$ which is needed to calculate the power of the interchannel interferences by equation (9) and then to calculate the probability of error by means of equation (10).

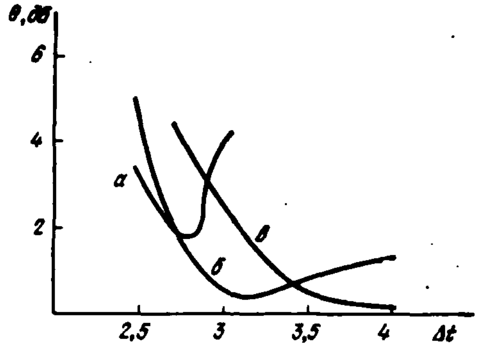

Fig. 4

Thus, if V = 32 kbauds Δf = 45 kHz for Batterworth filters $\Delta t_{опт} = 3{,}1$, then the total energy losses because of intersymbol and interchannel interference and uncoordinated filtration is equal to 2.4 dB for a probability of error 10^{-4}. For the same initial V and Δf for Chebyshev filters with $|K(j\omega)|_{мин} = -0{,}5$ дБ $\Delta t_{опт} = 2{,}75$, and the overall losses are equal to 3.4 dB.

The given example of calculating the energy losses shows that when simple filters are used it is advantageous to choose Batterworth filters.

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Synchronization of Coherent Receivers of Discrete Signals with Four Phase PM (OFM)

Yu. M. Braude-Zolotarev, V.M. Dorofeyev, and M.L. Payanskaya

The problem of synchronizing the cyclic and reference frequencies was studied in a coherent receiver of discrete PM signals (OFM) with phases 0, $\pi/2$, and $3\pi/2$.

Methods for minimizing the time of synchronization for a given freedom from interference under conditions of synchronizing with respect to the operating signal are studied.

The most complicated problem in the coherent reception of discrete signals is that of synchronizing the receiver with respect to the reference and cyclic frequencies. This complexity increases if the synchronization is done with respect to the operating signal and it requires a great accuracy for restoring the phases and rapid synchronization under conditions of a large frequency indefiniteness.

In the PM-4 system with phases $(0, +\pi/2, \pi, -\pi/2)$ the $(0, \pi)$ or $(-\pi/2, +\pi/2)$ conversions are possible with the simultaneous commutation of the phase of the orthogonal components. For these conversions the amplitude of the signal passes through zero and the phase is interrupted. Under such conditions it is relatively simple to separate the reference (carrier) frequency f_0 by quadrupling followed by filtering out the frequency $4f_0$. Then the cyclic frequency f_t can be separated from the signal of the given frequencies [1].

The PM-4 system with alternating commutation of the phases for the orthogonal components has been most widely used in satellite

communications lines. It has the lowest peak factor and it gives the higher freedom from interference for the receiver, and has other advantages [2, 3]. However, the synchronization in these systems is much more complicated since the resulting vector in neighboring cyclic intervals can change its position by no more than 90°. As a result of filtering the signal in transmission and receiving, its shape becomes close to that of an analog FM signal with a small modulation index ($m \simeq 1$). The instantaneous frequency $f \simeq 2\pi d\varphi/dt$ at the input to the demodulator can change in the range from $f_0 + R/4$ to $f_0 - R/4$ in which R (bits) is the total rate of transmission of the data by the signal with PM-4. These frequency changes are carried out smoothly without phase interruption and without the envelope of the signal's amplitude passing through zero (Fig. 1 b, d). It is apparent that in this case quadrupling the instantaneous frequency f_0 only results in a change in the modulation index which does not facilitate the problem of synchronization at all.

Under such conditions it is expedient to carry out the synchronization of the generator of the cyclic frequency (GT) directly in reference to the operating signal and independently of the synchronization of the generator of the reference [base] frequency (GO). The component f_T can be separated from the envelope for the signal's amplitude A(t) by the PM-4 amplitude detector AD (Fig. 1a) at the moment of time when the instantaneous frequency is established $f_0 + R/4$ or $f_0 - R/4$, since the signal from the PM-4 acquires an attendant amplitude modulation with a depth of about 20-30% (Fig. 1 b, c). The component f_T can be separated also (Fig. 1 a, d, e, f) from the pulses of the rectified derivative of the instantaneous frequency of the input signal [4].

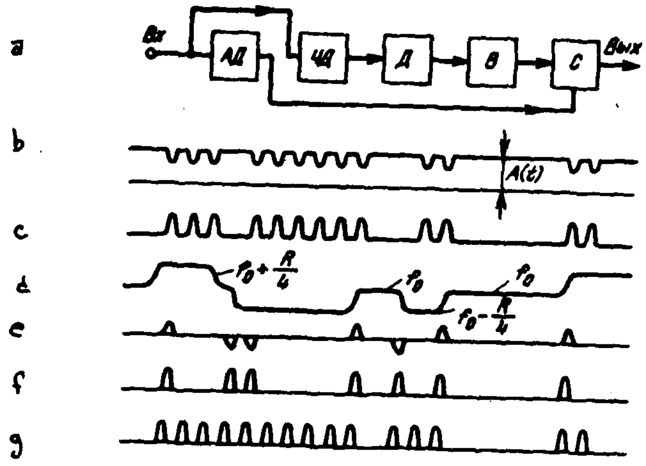

Fig. 1

b) amplitude envelope

c) pulses in branch AS)

d) signal ChD

e) differential ChD signal

f) pulses of cyclic synchronization in branch ChD

At the output of the frequency detector (ChD) a signal of different polarity is formed (Figs. 1, 2). After the differentiating filter (D) at the moment the instantaneous frequency changes by ±R/4 pulses of different polarity are formed df/dt (Fig. 1, e) which, after rectification (V) acquire the form $|df/dt|$ which is required for synchronizing the GT (Fig. 1f). In the former of the cyclic pulses (FTI), which is shown in Fig. 1, a, the pulses from AD and ChD are combined to improve the freedom from interference. Such synchronization of the GT using a single reading is the best since it is completely insensitive to interferences from the unsynchronized supporting frequency.

We note that at those moments when the carrier is transmitted there are no pulses of cyclic synchronization (Fig. 1g) and this makes it possible to separate the signal for synchronizing the GO directly in reference to the working signal by means of the method that was already mentioned of quadrupling the frequency.

However, this method is rarely used because it is not sufficiently interference-free.

Methods of synchronizing GO have been more widely used in which a demodulated signal of the data is used [2, 5-7]. According to Travin's method, the signal of the data removes the modulation from the input signal [6]. In this case 90° interruptions are introduced in the phase of the input signal. Therefore, this voltage is filtered out and then it is used to synchronize the GO. According to Iokoyama's method, the signal for the data forms a copy of the incoming signal in the discrete phase modulator PM-4 and filter. This copy and the input signal are fed into the phase detector in the circuit for synchronizing the GO [7]. Synchronization by means of a copy of the input signal combines best with the refined receivers which are designed for improving the freedom from interference.

Taking the foregoing into account, the version of synchronization shown in Fig. 2 is to be preferred. The frequency is converted in the mixer (Sm). By controlling the frequency of the generator for centering the spectrum, (GTS), tangential synchronization of the GO is assured. Since the frequency error is compensated for at the input to the band filter (Fl) stability is provided for its transition characteristics. It is obvious that for such synchronization the freedom from interference is higher than in cases in which the carrier frequency f_0 at the input to the filter is not stable.

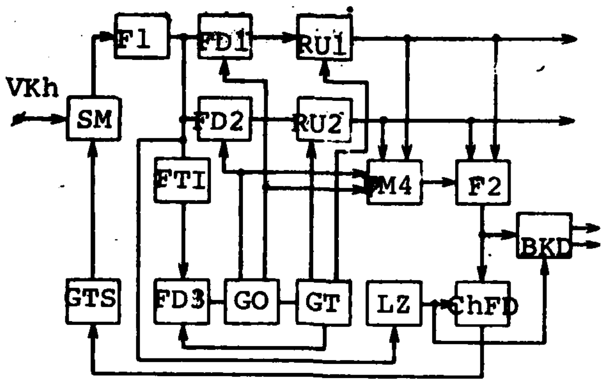

Fig. 2

A forming device for the cyclic pulses (FTI) is connected at the output to the filter. This gives the synchronization for the GT. The cyclic (read) pulses of the GT are formed by dividing the frequency of the GO which is achieved by the mutual synchronization of the OG and GT and the GT is synchronized by controlling the GO's frequency with respect to the FAPCh circuit in which a phase detector is connected FDZ. A FAPCh without filters is used because the initial indefiniteness for the cyclic frequency is very small and the permissible error for cyclic synchronization is of the order of 6° [8].

The phase detectors FD1 and FD2 form an evaluation of the phase of the input signal and the resolving device RU1 and RU2 form the resolution (single reading) as the read (cyclic) pulses from the GT come in. This resolution is used to form the copy of the input signal in the discrete phase modulator PM-4 and in the band filter F2.

This copy is fed onto one of the inputs of the frequency phase detector (ChFD) and it is fed into the other input of the ChFD from the filter F1 through the compensating delay

line (LZ) (Fig. 3). The operation of the block for correcting errors (BKO) does not apply directly to the problem of synchronization and will not be discussed here.

We know that the most significant hindrance to rapid synchronization is the lack of uniquiness for the characteristics of the phase detector (u_1 in Fig. 3), particularly under conditions for which the initial indefiniteness in the frequency $\Delta\omega_н$ is much greater than the frequency of the cut ω_c. The time for establishing it in an ideal second order FAPCh system with a filter K(p) = (p + a)/p, which has an unlimited capture band is approximately equal to [8]

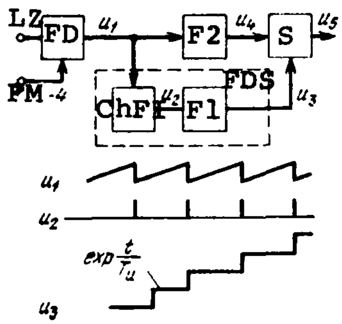

Fig. 3

$$t_{уст} \simeq \Delta\,\omega_н^2/a\,\omega_c^2. \qquad (1)$$

The time for establishment decreases when search is used in the same FAPCh system

$$t_{уст} \approx \Delta\omega_н/\omega_c^2. \qquad (2)$$

The time for establishment, determined by equations (1) and (2) is unusually large. A number of refined FAPCh have been described in the literature which can be used to reduce the time of establishment either by excluding the increment regions of the FD [9] or by converting the increment regions into decrement regions [10, 11] or by expanding the linear segment of the FD by N-fold (dividing the frequency of the signals by N [12]).

In the ChFD the time of establishment is decreased because of the use of a pulsed, frequency detector (ChDI) which forms a pulse (Fig. 3) at the moment the continuity of the FD's characteristics are interrupted. The filter-integrator (FI) with the transmission characteristic $\dot{K}_{\text{н}}(p) = 1/(1 + pT_{\text{н}})$ forms a voltage equivalent to the phase detector with a power characteristic (FDS). This integrator can be considered to be almost ideal if the conditions $\Delta\omega_{\text{ост}} \leqslant \omega_c$. From this it is necessary, for a ChFD, which is equivalent to a FAPCh without filters in which $\Delta\omega_{\text{ост}} = \Delta\omega_{\text{н}}/T_{\text{н}}\omega_c$, that

$$T_{\text{н}} > \Delta\omega_{\text{н}}/\omega_c^2. \quad (3)$$

For a FAPCh with a proportional integrating filter (PIF) $K_1(p) = (p+a)/(p+\varepsilon)$, where $\Delta\omega_{\text{ост}} \doteq \Delta\omega_{\text{н}}\varepsilon/T_{\text{н}}\omega_c a$ and usually $\approx 1/T_{\text{н}}$. From this we find

$$T_{\text{н}} > \Delta\omega_{\text{н}}\varepsilon/\omega_c^2 a, \quad (4a)$$

$$T_{\text{н}} \geqslant \omega_c^{-1}\sqrt{\Delta\omega_{\text{н}}/a}. \quad (4б)$$

Such parameters for the integrator can usually be realized in several ways if ω_c is not too small. In the case for which $\Delta\omega_{\text{н}}$ is large and ω_c is quite small, an adaptive filter must be used in the ChFD block or at its output. The filter's parameter is controlled automatically so that the condition $\varepsilon \simeq 0$ will be fulfilled [13].

By combining the characteristics of the FD and FDS (Fig. 3), the frequency-phase detector, can be assumed to be approximately like a FD with ideal linear characteristics within the limitations of the linear segment of the ChDI. For the assumptions that were made, a system for the automatic phase and frequency tuning (APChF) is described by the following operative equations:

for an APChF without a filter

$$\varphi(p) = \frac{1}{1 + \omega_c/p} \frac{\Delta \omega_н}{p^2}, \tag{5}$$

for an APChF with a PIF

$$\varphi(p) = \frac{1}{1 + \omega_c(a + p)/(\varepsilon + p)p} \frac{\Delta \omega_н}{p^2}. \tag{6}$$

By solving equation (5) we find that the time of establishment up to a value of the residual error $\varphi_{ост}$ is equal to

$$t_{уст} \approx \frac{1}{\omega_c} \ln\left[\frac{\Delta \omega_н}{\omega_c \varphi_{ост}}\right]. \tag{7}$$

The solution to equation (6) for PIF for which $\varepsilon < 10a$ shows that the optimum high speed is achieved for $a \simeq 0.25\omega_c$. In this case the time for establishment is equal to

$$t_{уст} = \frac{2}{\omega_c} \ln\left[\frac{2\Delta \omega_н}{\omega_c \varphi_{ост}}\right]. \tag{8}$$

In synchronizing the GO it is expedient to use an APChF without filters before capture, i.e., on the segment for which $\Delta\omega > \omega_c$ and an APChF with a PIF on the segment $\Delta\omega_c \leqslant \omega_c$, for which there is automatic tuning of the phases.

In calculating the freedom from interference for an APChF in the linear approximation, the results given in the literature [8] can be used.

The intrinsic apparatus looses in the selected system for synchronization in which the signal with the noise is multiplied once can be ignored. The residual phase error in using a PIF with $\varepsilon \ll a$, for any initial detunings $\Delta\omega_н$ is almost equal to zero and its mean square fluctuation in the synchronization channel is equal to [8]

$$\sigma_\varphi = 1/\sqrt{h_c} = \sqrt{N_0 F_c/p}\ , \qquad (9)$$

where h_c is the signal/noise ratio in the band of the synchronization system F_c, N_0 is the spectal density of the fluctuation noise, and p is the power of the received signal.

In order to get a mean square phase error of $\sigma_\varphi \simeq 1-2°$, a signal/noise ratio of about 30-36 dB must be provided in the synchronization circuit. In the channel for receiving the signal of the PM-4 the signal/noise ratio is usually $h_k \simeq$ 13 dB. Therefore, the noise band in the synchronization channel should be approximately equal to 0.01 of the noise band in the reception channel. For such conditions and for an initial detuning of the carrier by 10% of the channel's band width the time for establishment in a system without a filter t_{YCT} reaches 200-300 bits. Therefore, it is expedient to use the wider noise band of the APChF for frequency synchronization.

Thus, the synchronization of coherent PM-4 receivers with alternate commutation of the phase for the orthogonal components can be done with respect to the working signals. The speed and freedom from interference of the synchronization system can be increased greatly by carrying out the independent tuning with respect to the carrier and cyclic frequency by using a frequency phase detector with an adaptive filter in the APFCh's circuit and by using a system with a variable structure at the stages of sunchronization and holding.

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